Tutorial 1: Solution

1. An image is processed, and the following data extracted: $[s_1, d_2, f_3]$. Calculate the λ evidence that is propagated and the probability distributions over C.

$\lambda(e_1) = P(s_1|e_1)P(d_2|e_1) = 0$
$\lambda(e_2) = P(s_1|e_2)P(d_2|e_2) = 0.33 \times 0.33 = 0.11$
$\lambda(e_3) = P(s_1|e_3)P(d_2|e_3) = 0.14 \times 0.14 = 0.02$
and to propagate to C we need the conditioning equation
$\lambda(c_1) = (0 + 0.11P(e_2|c_1) + 0.02P(e_3|c_1)) \times P(f_3|c_1) = (0.11 \times 0.25 + 0.02 \times 0.25)0.125 = .004$
$\lambda(c_2) = (0 + 0.11P(e_2|c_2) + 0.02P(e_3|c_2)) \times P(f_3|c_2) = (0.11 \times 0.14 + 0.02 \times 0.72)0.14 = .0042$
Normalising over the evidence we have that $P'(C) = [0.488, 0.512]$ suggesting that the image is not a cat, but with no great certainty. (NB since the prior probabilities for c_1 and c_2 are the same we don't need to bother with them.

2. The same image is processed in monochrome. Thus there is no information for F. Re-calculate the probabilities of C.

Since there is no evidence from F, using the conditioning equation gives us that: $\lambda_F(c_1) = \lambda_F(c_2) = 1$ in other words we just consider the λ evidence from E. For: $[s_1, d_2]$
$\lambda(c_1) = (0 + 0.11P(e_2|c_1) + 0.02P(e_3|c_1)) = (0.11 \times 0.25 + 0.02 \times 0.25) = .0325$
$\lambda(c_2) = (0 + 0.11P(e_2|c_2) + 0.02P(e_3|c_2)) = (0.11 \times 0.14 + 0.02 \times 0.72) = .0298$
This time the network just favours the cat.

3. Doubt is expressed about the accuracy of the computer vision algorithms. Thus instead S and D are instantiated with virtual evidence. There is still no information for node F. Re-calculate the distributions over E and C.

This time its the evidence for E that changes.

$$
\begin{aligned}
\lambda(e_1) &= (0.8 \times P(s_1|e_1) + 0.2 \times P(s_2|e_1) \times (0.3 \times P(d_1|e_1) + 0.4 \times P(d_2|e_1) + 0.3 \times P(d_3|e_1)) \\
&= (0 + 0.2 \times 0.6) \times (0.3 \times 0.4 + 0.4 \times 0.4 + 0.2 \times 0.3) = 0.12 \times 0.34 = 0.041 \\
\lambda(e_2) &= (0.8 \times P(s_1|e_2) + 0.2 \times P(s_2|e_2) \times (0.3 \times P(d_1|e_2) + 0.4 \times P(d_2|e_2) + 0.3 \times P(d_3|e_2)) \\
&= (0.8 \times 0.33 + 0) \times (0.3 \times 0.33 + 0.4 \times 0.33 + 0.3 \times 0.34) = 0.264 \times 0.33 = 0.087 \\
\lambda(e_3) &= (0.8 \times P(s_1|e_3) + 0.2 \times P(s_2|e_3) \times (0.3 \times P(d_1|e_3) + 0.4 \times P(d_2|e_3) + 0.3 \times P(d_3|e_3)) \\
&= (0.8 \times 0.14 + 0.2 \times .14) \times (0.3 \times 0.29 + 0.4 \times 0.14 + 0.3 \times 0.14) = 0.0259
\end{aligned}
$$

since there is no evidence from F this is all that we need to propagate to C.
$\lambda(c_1) = (0.041P(e1|c_1) + 0.087P(e_2|c_1) + 0.0259P(e_3|c_1)) = (0.041 \times 0.5 + 0.087 \times 0.25 + 0.0259 \times 0.25) = .049$
$\lambda(c_2) = (0.041P(e1|c_2) + 0.087P(e_2|c_2) + 0.0259P(e_3|c_2)) = (0.041 \times 0.14 + 0.087 \times 0.14 + 0.0259 \times 0.72) = .037$
Normalising now gives us $P(C) = [0.57, 0.43]$

4. Given the evidence for C in part 3, calculate the probability distribution over F.

P(C) does not contain any evidence from F, so:

$$
\pi(F) = \begin{bmatrix} 0 & 0.3 \\ 0.125 & 0 \\ 0.125 & 0.14 \\ 0.25 & 0.14 \\ 0.125 & 0 \\ 0.125 & 0 \\ 0 & 0.14 \\ 0.125 & 0.14 \\ 0 & 0.14 \\ 0.125 & 0 \end{bmatrix} \begin{bmatrix} 0.57 \\ 0.43 \end{bmatrix} = \begin{bmatrix} 0.129 \\ 0.071 \\ 0.131 \\ 0.203 \\ 0.071 \\ 0.071 \\ 0.060 \\ 0.131 \\ 0.060 \\ 0.071 \end{bmatrix}
$$

The result does not need to be normalised in this particular example.