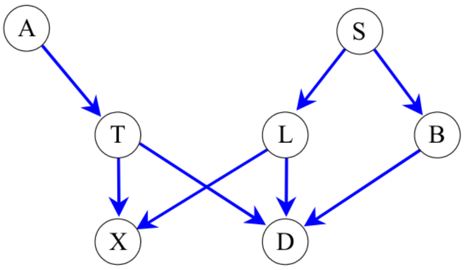
Tutorial 2: Solution

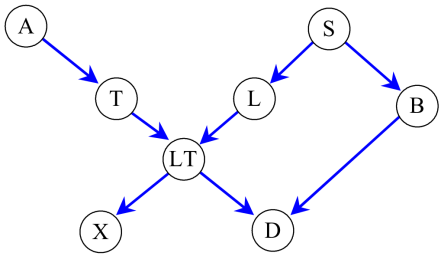
1(a):



A	Asia
B	Bronchitis
D	Dyspnea
L	Lung Cancer
S	Smoking
T	Tuberculosis
X	XRay

1(b): A problem occurs if either D or both D and X are instantiated. If D is instantiated there is a loop $D \to B \to S \to L$, and if X is instantiated as well there is another loop $D \to L \to X \to T$. L is the only node that belongs to both these loops, so if it is instantiated they are broken and any new propagation will terminate.

An interesting suggestion is made by Neopolitan. The network can be simplified by the introduction of an extra node LT meaning 'lung cancer of tuberculosis'. This removes one of the loops.



2a:
$\pi(e_1) = P(e_1|c_2) = 0.14$
$\pi(e_2) = P(e_2|c_2) = 0.14$
$\pi(e_3) = P(e_3|c_2) = 0.72$
2b:
$\lambda_F(c_1) = P(f_4|c_1) = 0.25$
$\lambda_F(c_2) = P(f_4|c_2) = 0.14$
Evidence for C excluding E (the π message from C to E) is:
$\pi_E(c_1) = 0.25 * 0.6 = 0.15(P'(c_1) = 0.73)$
$\pi_E(c_2) = 0.14 * 0.4 = 0.056(P'(c_2) = 0.27)$
Hence:

$$\pi(E) = \begin{bmatrix} 0.5 & 0.14 \\ 0.25 & 0.14 \\ 0.25 & 0.72 \end{bmatrix} \begin{bmatrix} 0.73 \\ 0.27 \end{bmatrix} = [0.4, 0.22, 0.38]$$

Q3: The data frequencies are:

	$c_1\&w_1$	$c_1\&w_2$	$c_2\&w_1$	$c_2\&w_2$
e_1	0	4	1	0
e_2	0	2	1	0
e_3	0	2	1	4

hence the probabilities in the link matrix are:

	$c_1\&w_1$	$c_1\&w_2$	$c_2\&w_1$	$c_2\&w_2$
e_1	0.33	0.5	0.33	0
e_2	0.33	0.25	0.33	0
e_3	0.34	0.25	0.34	1

Q4
$\pi_E(W) = [0, 1]$
$\pi_E(C) = [0.73.0.27]$ (from part 2)
$\pi_E(c_1 \& w_1) = 0$
$\pi_E(c_1 \& w_2) = 0.73$
$\pi_E(c_2 \& w_1) = 0$
$\pi_E(c_2 \& w_2) = 0.27$
Multiplying the link matrix by the joint probabilities gives:
$\pi(e_1) = 0.73 \times 0.5 = 0.365$
$\pi(e_2) = 0.73 \times 0.25 = 0.18$
$\pi(e_3) = 0.73 \times 0.25 + 0.27 \times 1 = 0.45$
The total evidence for E is found by multiplying the λ and π evidence together and it is normalised to find $P'(E)$.

One way to find the λ evidence sent from E to C is to consider reducing the martix $P(E|C\&W)$ to $P(E|C)$ for the specific case given the evidence for W. This is calculated as an average weighted by the evidence for W:
$P(e_1|c_1) = P(e_1|c_1\&w_1) \times \pi_E(w_1) + P(e_1|c_1\&w_2) \times \pi_E(w_2)$
Since we have that $\pi_E(w_1) = 0$ and $\pi_E(w_1) = 1$ it follows that:
$P(e_1|c_1) = P(e_1|c_1\&w_2)$
etc.

There is no need to normalise so we can say:
$\lambda_E(c_1) = P(e_1|c_1)\lambda(e_1) + P(e_2|c_1)\lambda(e_2) + P(e_3|c_1)\lambda(e_3) = 0.5 \times 0.5 + 0.25 \times 0.3 + 0.25 \times 0.2 = 0.375$
$\lambda_E(c_2) = P(e_1|c_2)\lambda(e_1) + P(e_2|c_2)\lambda(e_2) + P(e_3|c_2)\lambda(e_3) = 1 \times 0.2 = 0.2$

Thus the total evidence for C is:
$\epsilon(c_1) = 0.375 \times 0.15 = 0.056$
$\epsilon(c_2) = 0.2 \times 0.056 = 0.0112$
Normalising we get:
$P'(c_1) = 0.833$
$P'(c_2) = 0.167$

Notice how our knowledge of there not being an owl increases our belief that there is a cat.