Tutorial 3: Solution

1. (a) The following happens during initialisation:
   - A sends a π message to T, S sends a π message to L and B
   - T,L and B recalculate their posterior probability
   - T sends a π message to X and D, L sends a π message to X and D, B sends a π message to D.
   - X and D recalculate their posterior probabilities (with λ evidence 1 for each state)
   - Propagation finishes

   (b) S sends a π message to L and B over-riding previous message based on its prior probability.
   - L and B re-calculate their posterior probability (with λ evidence 1 for each state)
   - L sends a π message to X and D, B sends a π message to D
   - X and D recalculate their posterior probabilities (with λ evidence 1 for each state)
   - Propagation finishes

   (c) X sends a λ message to T and L
   - T and L calculate their posterior probability from both λ and π evidence.
   - T sends a λ message to A and a π message to D
   - A and D recalculate their posterior probabilities. (D still has no λ evidence)
   - L sends a λ message to S (which is ignored) and a π message to D.
   - D recalculates its posterior probability distribution (D still has no λ evidence)
   - Propagation finishes

   (d) The problem now is that a loop is established. After instantiation D (among other things) sends a λ message to T which sends a π message to X which sends a λ message to L which sends a π message to D, and probability propagation never terminates.

2. T has two states t_1 means tuberculosis present and t_2 tuberculosis not present. Hence:
   $P(lt_1|t_1\&l_1) = P(lt_1|t_1\&l_2) = P(lt_1|t_2\&l_1) = 1$
   $P(lt_1|t_2\&l_2) = 0$
   The columns of the link matrix sum to 1, hence we can write it as:

$$P(LT|T\&L) = \begin{bmatrix} P(lt_1|t_1\&l_1) & P(lt_1|t_1\&l_2) & P(lt_1|t_2\&l_1) & P(lt_1|t_2\&l_2) \\ P(lt_2|t_1\&l_1) & P(lt_2|t_1\&l_2) & P(lt_2|t_2\&l_1) & P(lt_2|t_2\&l_2) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. The first step is to calculate the π evidence for LT (operating equation 4). We can write $\pi_{LT}(t_1\&l_1) = \pi_{LT}(t_1) \times \pi_{LT}(l_1) = 0.2$. Similarly $\pi_{LT}(t_1\&l_2) = 0.2, \pi_{LT}(t_2\&l_1) = 0.3, \pi_{LT}(t_2\&l_2) = 0.3$. Multiplying this by the link matrix we get $\pi(lt_1) = 0.7$,and$\pi(lt_2) = 0.3$. Amassing all the evidence at LT we get $\epsilon(LT) = (0.21, 0.21)$ and the probability distribution is therefore $(0.5, 0.5)$.

   Now tuberculosis is instantiated to state t_2 so $\pi_{LT}(T)$ becomes $(0, 1)$. So, $\pi_{LT}(t_1\&l_1) = 0, \pi_{LT}(t_1\&l_2) = 0, \pi_{LT}(t_2\&l_1) = 0.5, \pi_{LT}(t_2\&l_2) = 0.5$ and hence $\pi(lt_1) = \pi(lt_2) = 0.5$. The total evidence for LT is now and $(0.15, 0.35)$ and $P'(LT) = (0.3, 0.7)$.

   We can now calculate the π messages to the children. $\pi_X(LT) = P'(LT)/\lambda_X(LT) = (1, 1)$ (operating equation 2) (notice that because the π evidence for LT happens to be $(0.5, 0.5)$ the only evidence for one state or the other of LT is the λ evidence from X, hence the π message to X correctly conveys no evidence.)
   $\pi_D(LT) = P'(LT)/\lambda_D(LT) = (0.3, 0.7)$.
   The λ message sent to L is calculated using operating equation 1.
   $\lambda_{LT}(l_1) = 0 \times (1 \times 0.3 + 0 \times 0.7) + 1 \times (1 \times 0.3 + 0 \times 0.7) = 0.3$
   $\lambda_{LT}(l_2) = 0 \times (1 \times 0.3 + 0 \times 0.7) + 1 \times (0 \times 0.3 + 1 \times 0.7) = 0.7$

4. If D is not instantiated then propagation will terminate for any other instantiation. If D is instantiated, then one of LT, L, S or B must also be instantiated for the propagation to terminate.