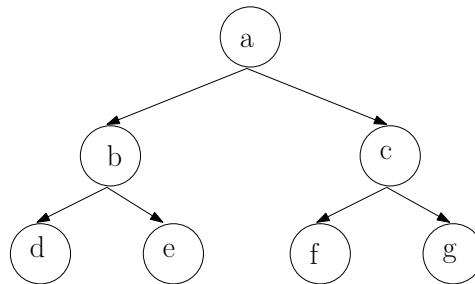**Figure 1:** Directed graphical model.

Figure 2: Directed Tree.

1. Given the graphical model in Figure 1, which of the following conditional independence statements are correct?

   (a) $a \perp\!\!\!\perp f$ false

   (b) $a \perp\!\!\!\perp g$ true

   (c) $b \perp\!\!\!\perp i|f$ false

   (d) $d \perp\!\!\!\perp j|g,h$ true

   (e) $i \perp\!\!\!\perp b|h$ true

   (f) $j \perp\!\!\!\perp d$ false

   (g) $i \perp\!\!\!\perp c|h,f$ false

2. Let us consider the following directed graph in Figure 2.

   (a) Find the undirected graph that encodes the same conditional independences as the directed graph.
   The solution is given in Figure 3.

   (b) Find the joint-tree of the undirected graph.
   The solution is given in Figure 4.
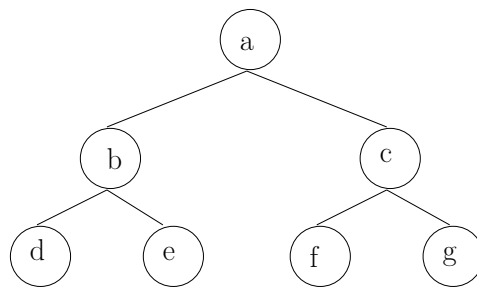
Figure 3: Undirected Tree.
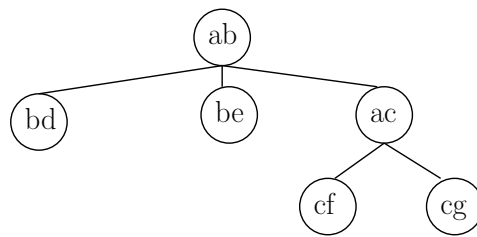
Figure 4: Joint Tree.

(c) What is the joint probability distribution that encodes these conditional independences.

The joint probability distribution is given by

$$p(\pmb{x}) = \frac{p(x_{ab})p(x_{bd})p(x_{be})p(x_{ac})p(x_{cf})p(x_{cg})}{p(x_a)p(x_b)^2 p(x_c)^2}$$