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Anisotropic Mesh Adaptivity on nVIDIA's CUDA

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Introduction

- Computational Fluid Dynamics (CFD)
  - Study of the motion of fluids
  - Modelled using the Navier-Stokes partial differential equations (PDEs)
  - Usually not possible to get analytical solutions
  - Numerical methods
- Finite Element Method (FEM)
  - The domain (i.e. the surface or volume) of interest and the PDE are discretised into sub-domains and the PDE is solved inside each one
  - Sub-domains are elements, usually of triangular (2D) or tetrahedral (3D) shape, which form a structure referred to as mesh.
  - Structured vs. unstructured meshes: tradeoff between computational complexity and numerical flexibility.

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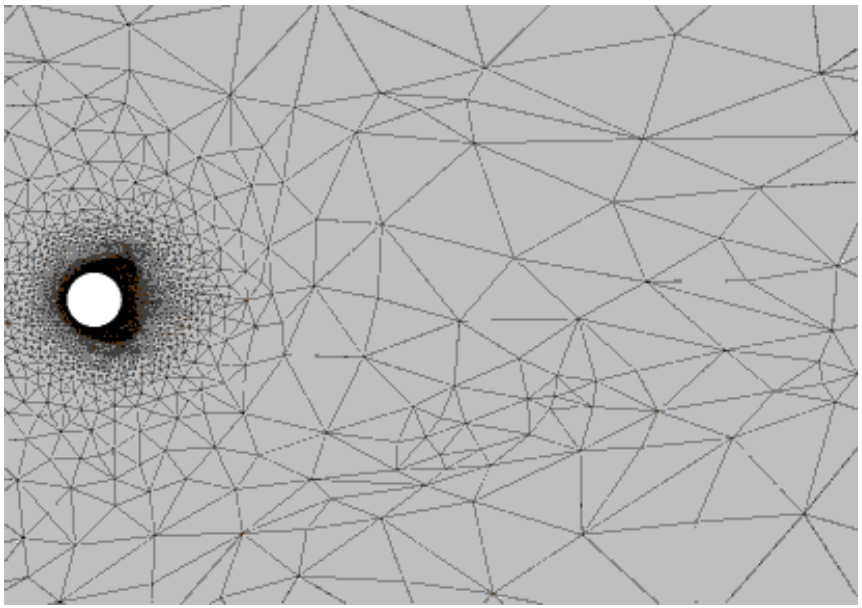
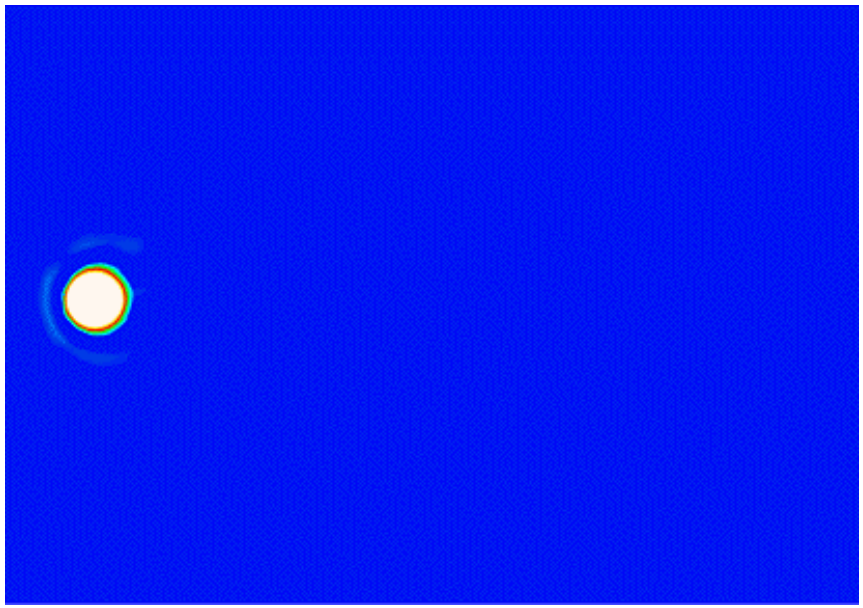
Introduction

- Example application: Mesh adaptivity following flow features past an infinite heated cylinder, simulating the pattern of vortices known as Kármán vortex street.





http://amcg.ese.ic.ac.uk/index.php?title=Heated_Cylinder_Adapt_Example

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Motivation

- Mesh adaptivity is a computationally heavy task
  - Lots of floating-point arithmetic
- Parallel versions of adaptivity algorithms exist
  - Help speed up the process using MPI, multi-core architectures, GPGPUs etc.
- Good candidate for GPGPU computing
  - Massively parallel architectures manipulating thousands of threads
  - Great floating-point capabilities
- Related work: Fluidity
  - Open source CFD solver, developed at the Applied Modelling and Computation Group, Imperial College London
  - Latest release (4.0) runs on conventional hardware

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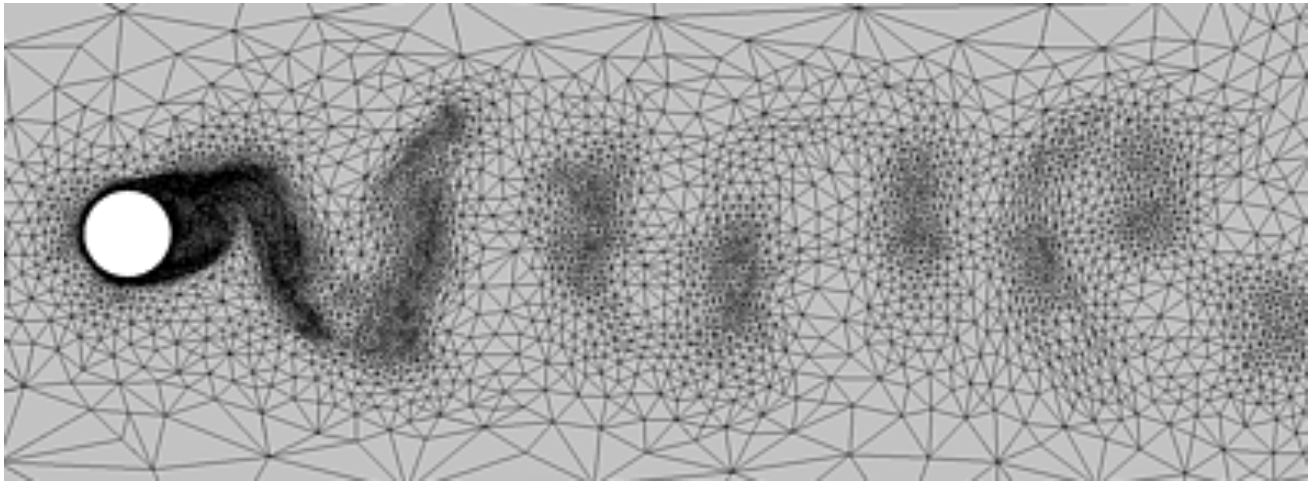
Unstructured Meshes

- No regular pattern of discretisation (in contrast to, e.g., a matrix)
- Need explicit information about vertex and element adjacency



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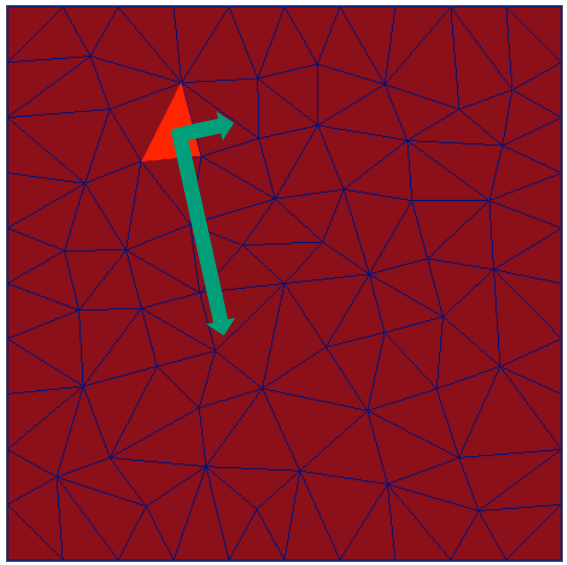
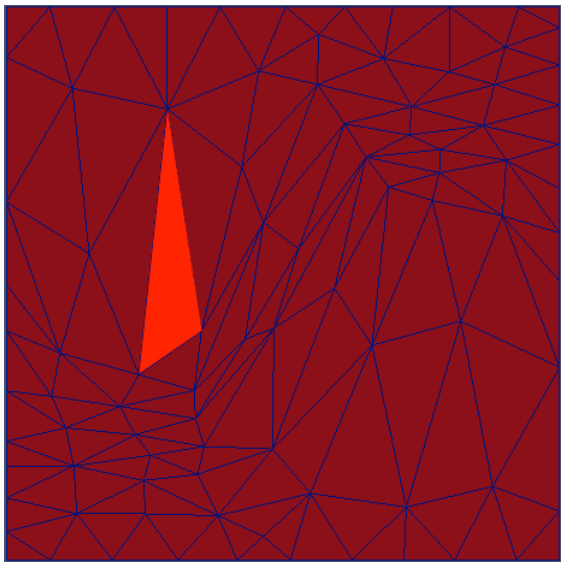
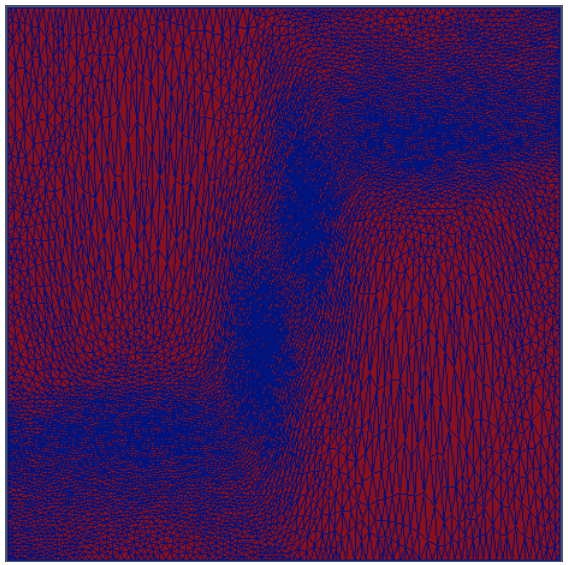
Anisotropic PDEs

- Directional dependencies: convergence speed and accuracy are favoured if mesh elements have a suitable orientation (size and shape).
- Metric tensor field: for each point, represents the desired length and orientation of a mesh edge containing this point
  - Can be decomposed as: $M = Q \Lambda Q^T$
    - » Q is an orthonormal matrix representing the rotation of the axis system so that the base vectors show the directions to which the element has to be stretched.
    - » Λ is a diagonal matrix representing the amount of stretching in each direction.







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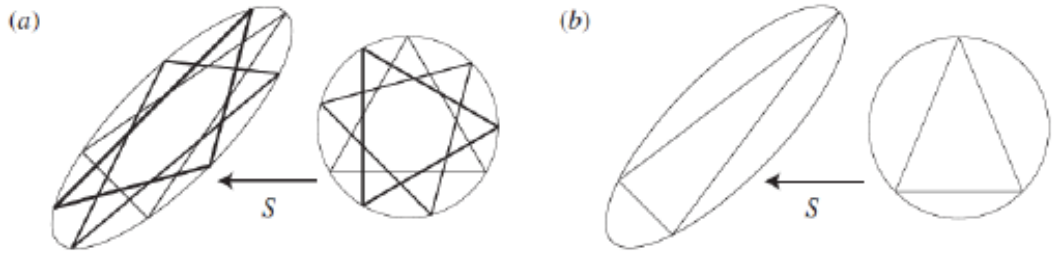
Metric tensor field and element quality

- In 2D space, it is a 2x2 matrix: $M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$
- Discretised vertex-wise.
- Vasilevskii & Lipnikov, 1999: the ideal triangle is an equilateral one with edges of unit length with respect to the metric tensor field.



"Anisotropic mesh adaptivity for multi-scale ocean modelling", Piggott, Farrell, Wilson, Gorman, Pain, 2009

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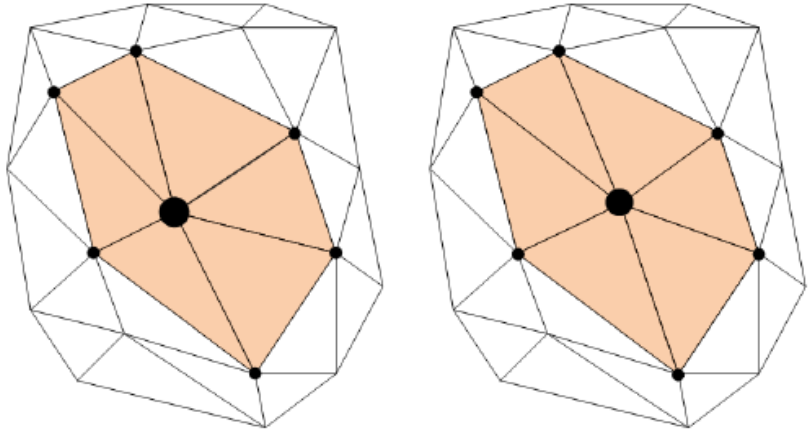
Vertex Smoothing

- A simple technique which does not affect mesh topology, i.e. number of elements and connectivity of vertices.
- Examines one mesh “Cavity” at a time, i.e. a central vertex and all its adjacent vertices.
- Central vertex is relocated to a new position such that the quality of the lowest-quality triangle is maximised.



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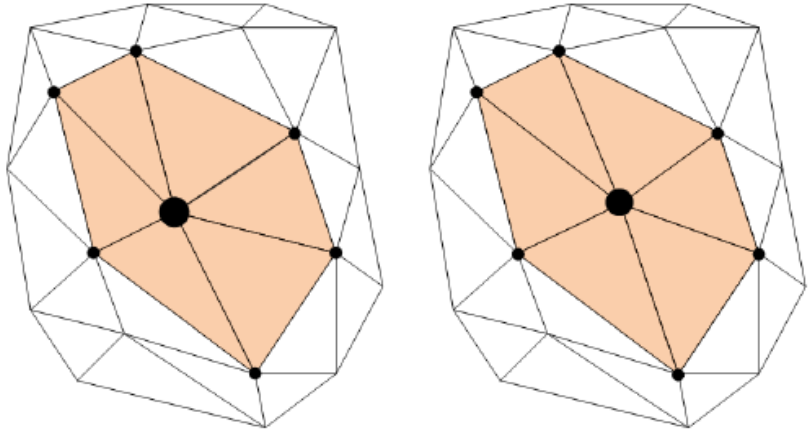
Propagation and Iterations

- The scope of optimisation is local, i.e. within the cavity.
- After neighbouring cavities have been processed, i.e. adjacent vertices have been relocated, quality of the first cavity may have been degraded.
- Need to perform multiple passes over the mesh in order to bring things to an equilibrium.
- As vertices are relocated to new positions, the metric tensor field for the new position is calculated by interpolating the metric tensor values at the nearest vertices.



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Algorithm by Pain et al.

- The new position x of the central vertex (with initial position p) with respect to the positions y^j of the adjacent vertices is obtained by solving the linear system:

$$(D + A)(x - p) = 0.5(q - Ap)$$

where

$$A = \sum_l M_l, \forall\ edge\ \ l\ \in cavity$$

$$q = \sum_l M_l y_l^j, \forall\ edge\ \ l\ \in cavity$$

and

$$D_{jk} = \begin{cases} \max\left(A_{jj}, 1.01 \sum_{m=1, m\neq j} |A_{jm}|\right), if\ j = k \\ 0, if\ j \neq k \end{cases}$$

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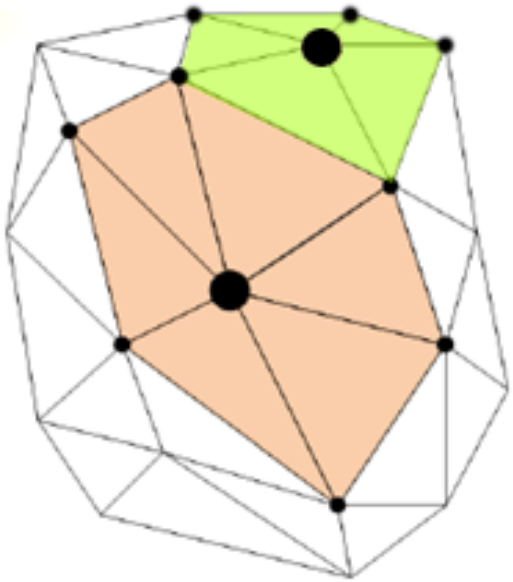
Parallel Execution

- Adjacent vertices cannot be relocated simultaneously because the new position of a vertex depends on the old positions of all its neighbouring vertices.
- Idea: mesh vertices are coloured so that no two adjacent vertices have the same colour.
- All vertices that have the same colour, i.e. belong to the same independent set, can be smoothed simultaneously.



Based on the framework by Freitag et al.

Implementation

- Array V of vertices, i.e. pairs of coordinates.
- Array C of cavities, i.e. sets of vertex indices of array V, such that the cavity defined by a vertex is accessed using the same index as the one used to access the vertex in V.
- 2D array containing the values of the metric tensor field:
  - The structured representation allows us to treat the metric tensor field as a graphics texture, taking advantage of specialised hardware.
  - In order to transform the unstructured representation to a structured one, it is necessary to super-sample the unstructured representation.
- For a pre-defined number of iterations over the mesh do:
  - Process one independent set (colour) at a time:
    - » Invoke the optimisation kernel, creating as many GPU threads as there are vertices in the set, grouping them in CUDA blocks according to the execution configuration.
    - » Each thread optimises exactly one cavity.
  - Until all independent sets have been processed.

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Metric Tensor Field as Graphics Texture

- Recall that metric tensor field values at new vertex positions are the result of interpolation.
- The same thing happens with graphics textures.
  - A texture is a collection of colour samples at various points.
  - Values at in-between points are calculated by interpolation.
- Benefits:
  - Interpolation is done by dedicated, specialised texture hardware.
    - » Computations in hardware may be completed faster than in software.
    - » CUDA execution units are free to accomplish other tasks.
    - » Fewer registers are used by the kernel, leading to better warp occupancy.
  - Textures are cached in a dedicated cache, leaving more room for vertex data in shared memory/L1/L2 caches.
- Drawbacks:
  - Super-sampling needs more memory.
  - Loss in accuracy is also possible, as mesh density in certain areas may be higher than the resolution a structured representation can offer.

Using dedicated texture hardware to do the interpolation

$$tex(x,y) = (1-\alpha)(1-\beta)T[i,j] + \alpha(1-\beta)T[i+1,j] + (1-\alpha)\beta T[i,j+1] + \alpha\beta T[i+1,j+1]$$

```
#if defined(USE_TEXTURE_MEMORY)
    float4 floatMetric = tex2D(metricTex, coords.iCoord, coords.jCoord);
#else
    double iIndex = coords.jCoord * meshSize;
    int i = floor(constant1 * iIndex);
    iIndex -= i;

    double jIndex = coords.iCoord * meshSize;
    int j = floor(constant1 * jIndex);
    jIndex -= j;

    if(i == constant3)        // if we are on the top or bottom boundary
            metric = cudaMetricValues[constant2 +   j  ] * (1 - jIndex) +
                     cudaMetricValues[constant2 + (j+1)] *    jIndex;
    else if(j == constant3) // if we are on the left or right boundary
            metric = cudaMetricValues[(i+1) * meshSize - 1] * (1 - iIndex) +
                     cudaMetricValues[(i+2) * meshSize - 1] *    iIndex;
    else
            metric = cudaMetricValues[  i   * meshSize +   j  ] * (1 - iIndex) * (1 - jIndex) +
                     cudaMetricValues[  i   * meshSize + (j+1)] * (1 - iIndex) *    jIndex    +
                     cudaMetricValues[(i+1) * meshSize +   j  ] *    iIndex    * (1 - jIndex) +
                     cudaMetricValues[(i+1) * meshSize + (j+1)] *    iIndex    *    jIndex;
#endif
```

Setting up texture memory

```
#if defined(USE_TEXTURE_MEMORY)
    cudaChannelFormatDesc channelDesc =
            cudaCreateChannelDesc(32, 32, 32, 32, cudaChannelFormatKindFloat);
    cudaMallocArray(&cudaMetricValues, &channelDesc, meshSize, meshSize);
    cudaMemcpyToArray(cudaMetricValues, 0, 0, floatMetricValues,
            meshSize * meshSize * sizeof(float4), cudaMemcpyHostToDevice);

    metricTex.normalized = true;
    metricTex.filterMode = cudaFilterModeLinear;
    metricTex.addressMode[0] = cudaAddressModeClamp;
    metricTex.addressMode[1] = cudaAddressModeClamp;

    cudaBindTextureToArray(metricTex, cudaMetricValues, channelDesc);
#else
    cudaMalloc((void **) &cudaMetricValues, meshSize * meshSize * sizeof(Vector2dPair));
    cudaMemcpy(cudaMetricValues, metricValues,
            meshSize * meshSize * sizeof(Vector2dPair), cudaMemcpyHostToDevice);
#endif
```

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Experiments

- Serial vs. 8-threaded OpenMP vs. CUDA
- Double-precision arithmetic
  - Apart from the metric tensor field – single precision is enough
- Workstation hosting:
  - Two Intel Clovertown quad-core Xeon X5355 (2.66GHz)
  - One Tesla C2050 graphics board (Fermi architecture).
  - Environment:
    - » Ubuntu Server, running kernel 2.6.32-24-server x86_64
    - » gcc 4.4.3
    - » CUDA SDK 3.1 and nVIDIA Compilation Tools 3.1, V0.2.1221
    - » nVIDIA ForceWare driver, version 256.40

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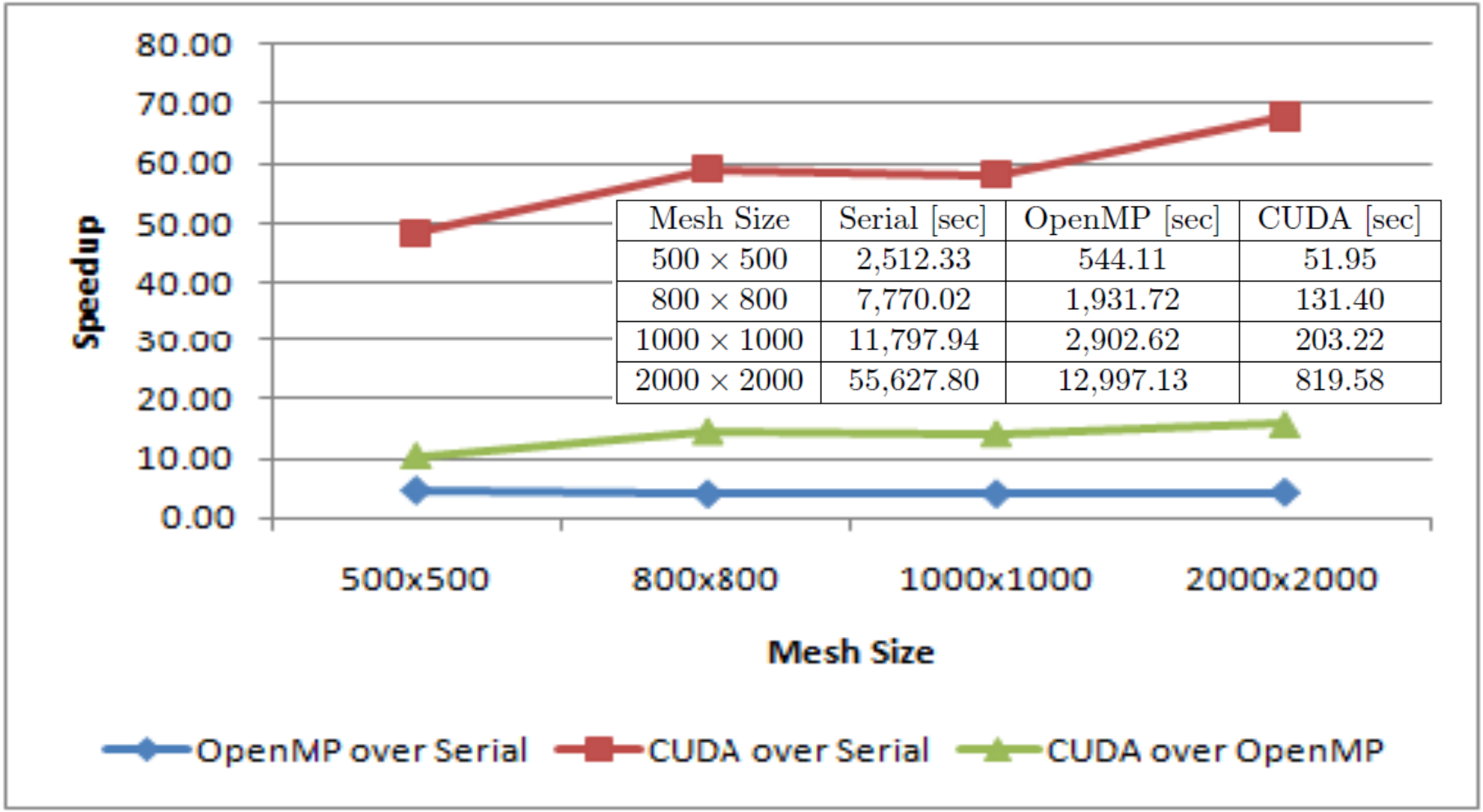
Experimental results

- Register usage and warp occupancy:
  - Structured metric tensor field – non-textured: 59 registers, 33.3% occupancy
  - Structured metric tensor field – textured: 51 registers, 37.5% occupancy
  - In both cases, occupancy is noticeably high
    - » Leaving as future work the break-down of the CUDA kernel into smaller parts.
- Best execution configuration was found to be:
  - 32 threads per CUDA block for the non-textured version.
  - 16 threads per CUDA block for the textured version.
  - Using more threads per block has a negative impact on performance, indicating that memory access latency is significant
    - » Leaving as future work the partitioning of the mesh in an effort to improve data locality.

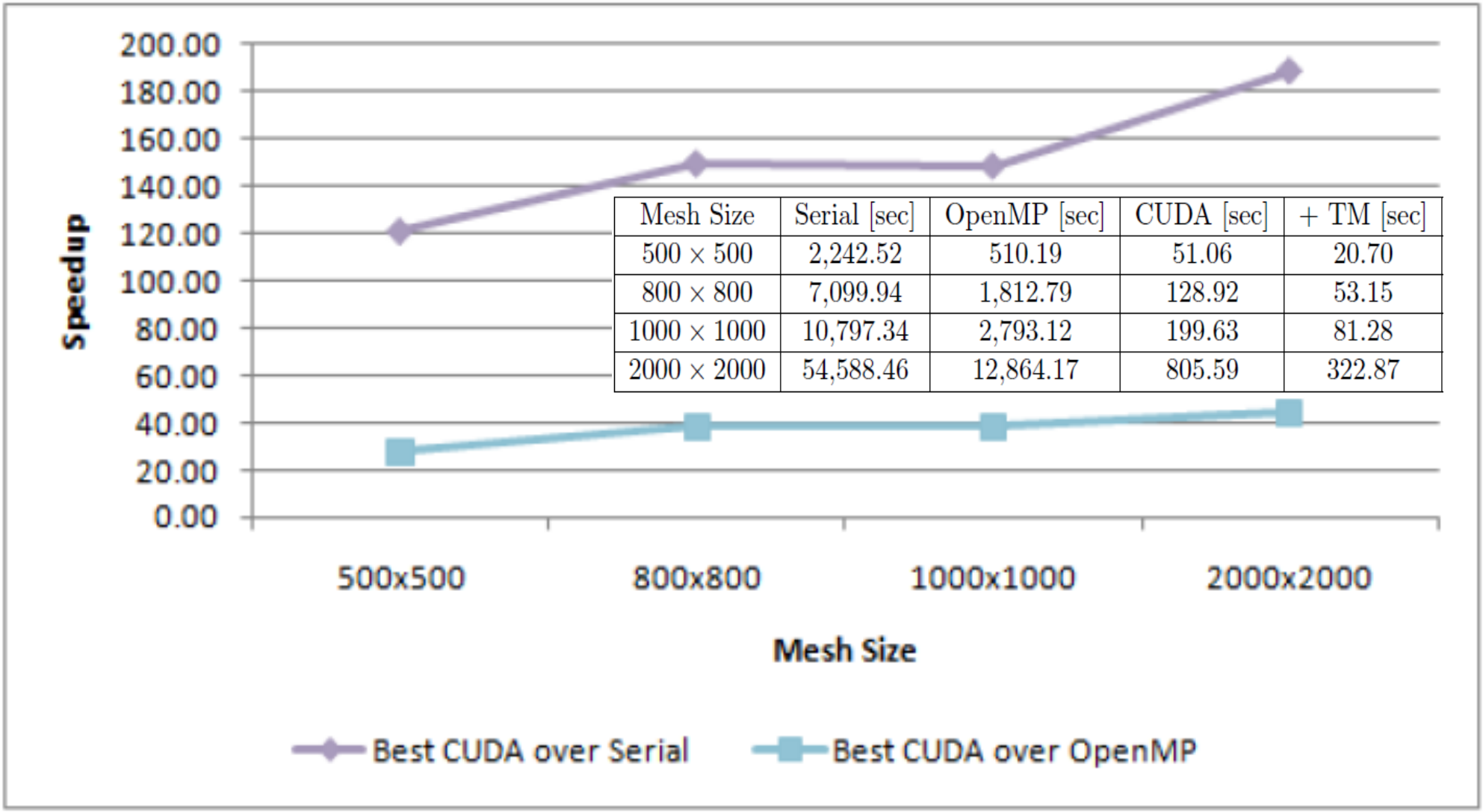
Speedup diagrams – non-textured approach



Mesh Size	Serial [sec]	OpenMP [sec]	CUDA [sec]
500 × 500	2,512.33	544.11	51.95
800 × 800	7,770.02	1,931.72	131.40
1000 × 1000	11,797.94	2,902.62	203.22
2000 × 2000	55,627.80	12,997.13	819.58

- Serial vs. 8-threaded OpenMP vs. non-textured CUDA
  - The 8-threaded OpenMP version is approx. x5 faster than the serial version
  - The CUDA version is up to x16 faster than the OpenMP version.
  - The CUDA version is up to x68 faster than the serial version.
    - » Highest speedups are observed when running really large problems.

Speedup diagrams – textured approach



Mesh Size	Serial [sec]	OpenMP [sec]	CUDA [sec]	+ TM [sec]
500 × 500	2,242.52	510.19	51.06	20.70
800 × 800	7,099.94	1,812.79	128.92	53.15
1000 × 1000	10,797.34	2,793.12	199.63	81.28
2000 × 2000	54,588.46	12,864.17	805.59	322.87

- Serial vs. 8-threaded OpenMP vs. textured CUDA
  - Using the texture approach speeds up the CUDA version by up to x2.5!
  - The optimized CUDA version is up to x45 faster than the OpenMP version.
  - The optimised CUDA version is up to x190 faster than the serial version.

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Conclusions and future work

- This first attempt to implement anisotropic mesh adaptivity on CUDA showed that great benefits can still be obtained, despite the unstructured nature of the problem.
- Speedup can be as high as x190 compared to the serial CPU code and up to x45 compared to an eight-threaded OpenMP code.
- Treating the metric tensor field as a graphics texture has more than doubled CUDA performance.
- Breakdown of this CUDA kernel into smaller parts and partitioning to improve data locality are left as future work.
- Porting the application to OpenCL would offer a testing framework to assess OpenCL compilers and also compare CUDA vs. Stream vs. CBE.
- Porting the codebase to OP2 to assess ease of coding and achieved performance (a task which is already in progress).
- The full work along with references to publications for further reading can be obtained from my web page:

http://www.doc.ic.ac.uk/~gr409/