Gadgets

A **gadget** is a partial register-machine graph.

It has one entry wire, and one or more exit wires.

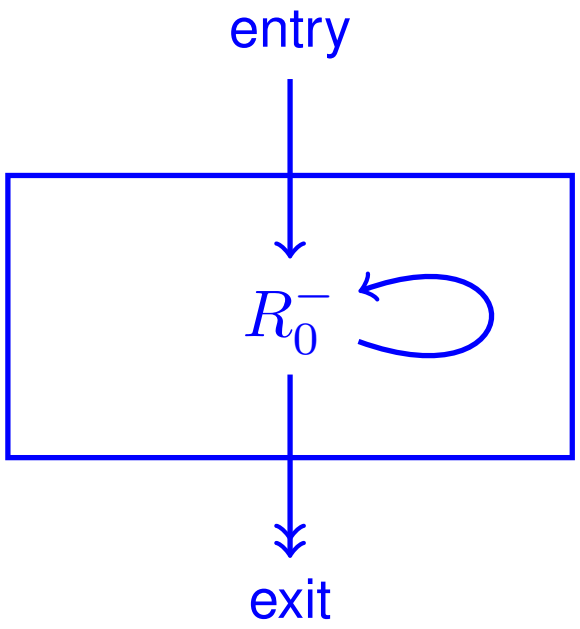
The gadget operates on input and output registers specified in the gadget's name.

The gadget may use other registers, called scratch registers, for temporary storage.

The gadget assumes the scratch registers are initially set to 0, and **must** ensure that they are set back to 0 when the gadget exits.

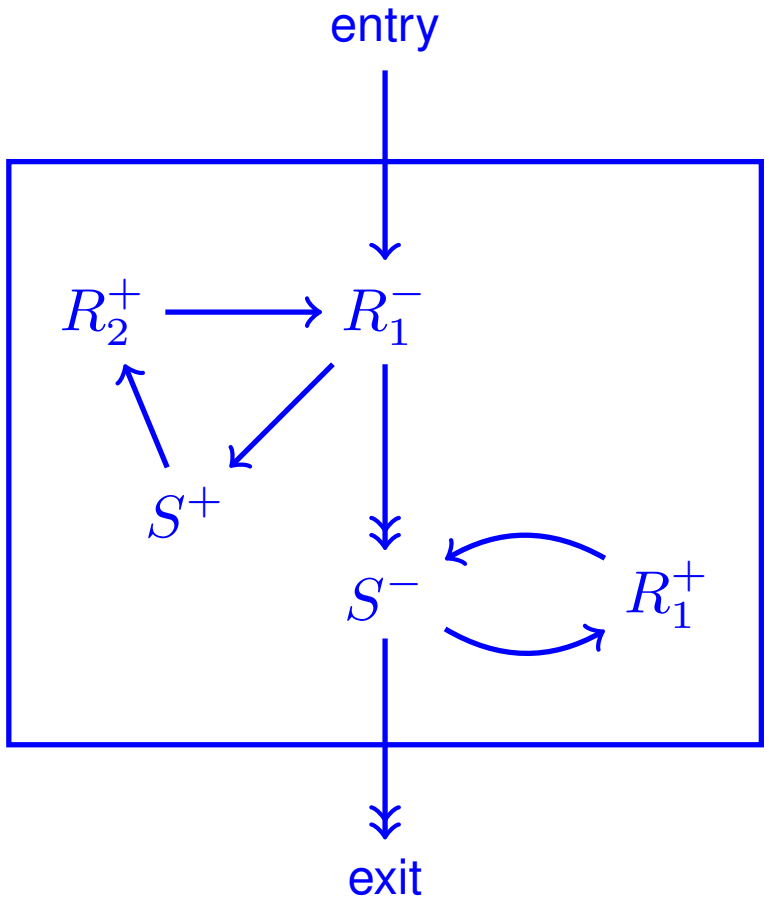
Gadget: "zero R_0"

The gadget "zero R_0" sets register R_0 to be zero, whatever its initial value:



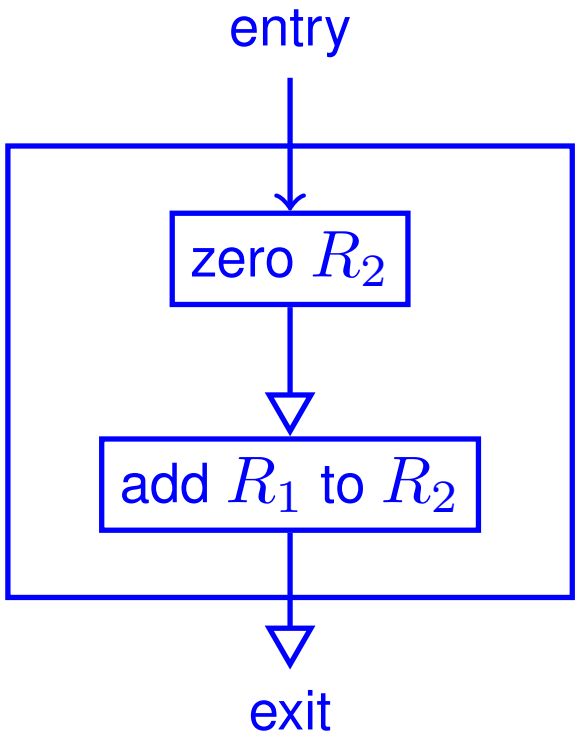
Gadget: "add R_1 to R_2"

The gadget "add R_1 to R_2" adds the initial value of R_1 to register R_2, storing the result in R_2 but restoring R_1 to its initial value.

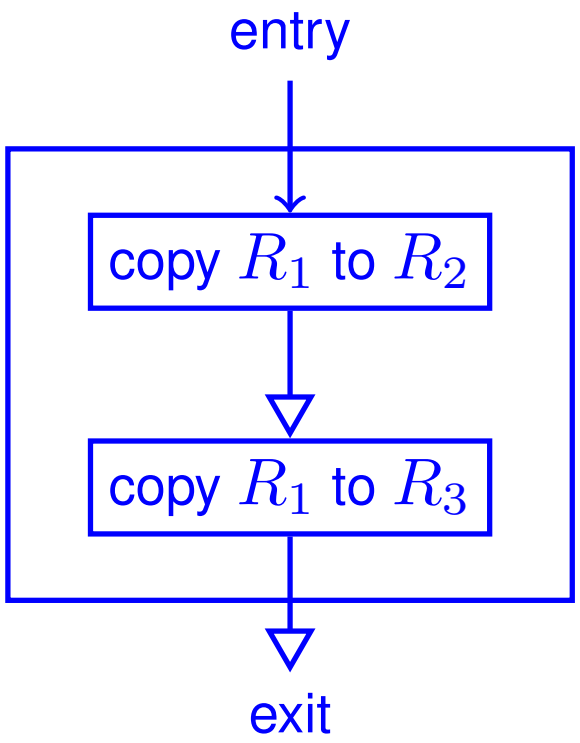


Gadget: "copy R_1 to R_2"

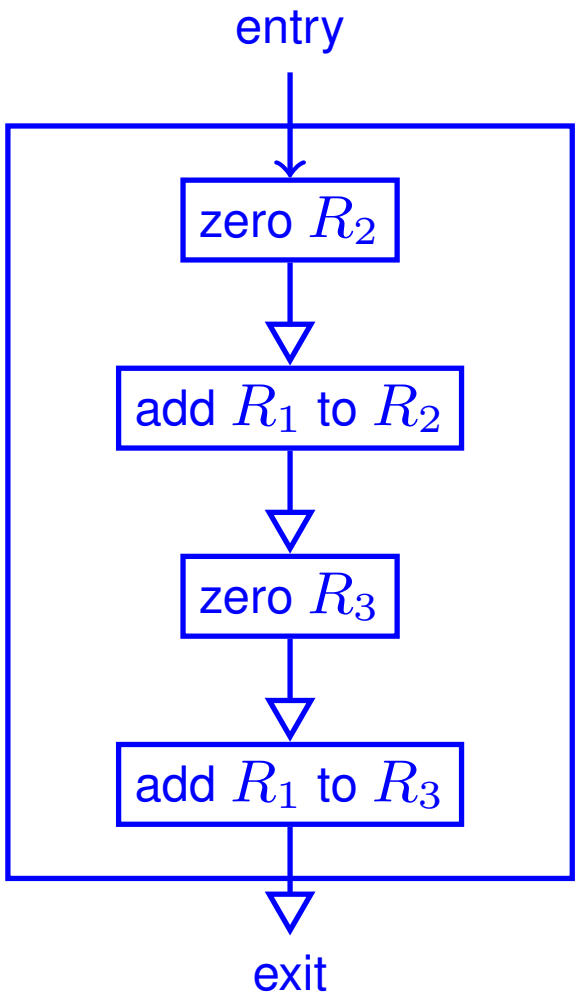
The gadget "copy R_1 to R_2" copies the value of register R_1 into register R_2, leaving R_1 with its initial value:



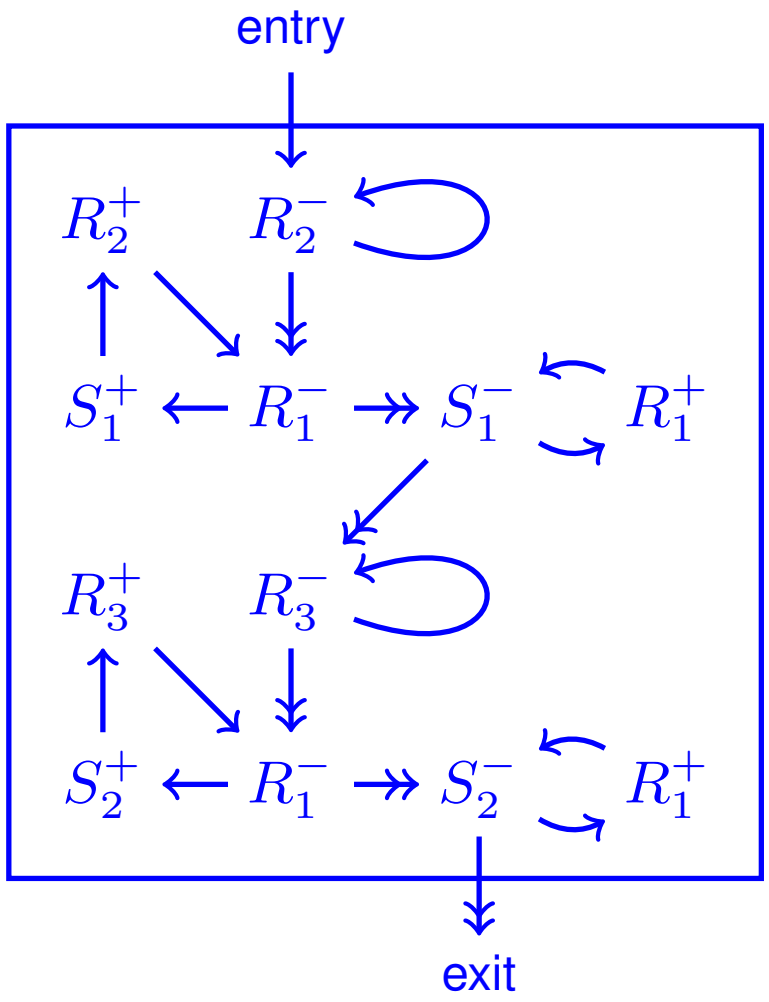
Gadget : "copy R_1 to R_2 and R_3"



Gadget: "copy R_1 to R_2 and R_3"

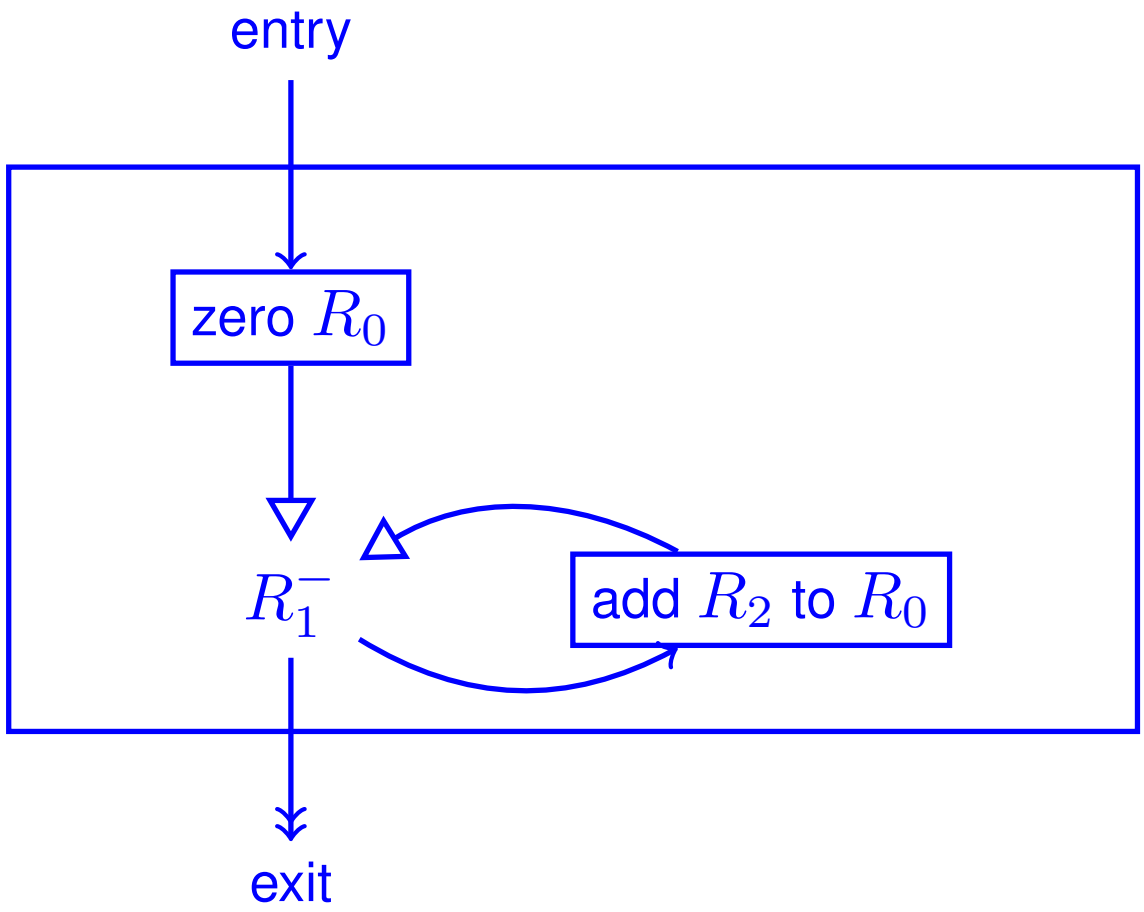


Gadget: "copy R_1 to R_2 and R_3"



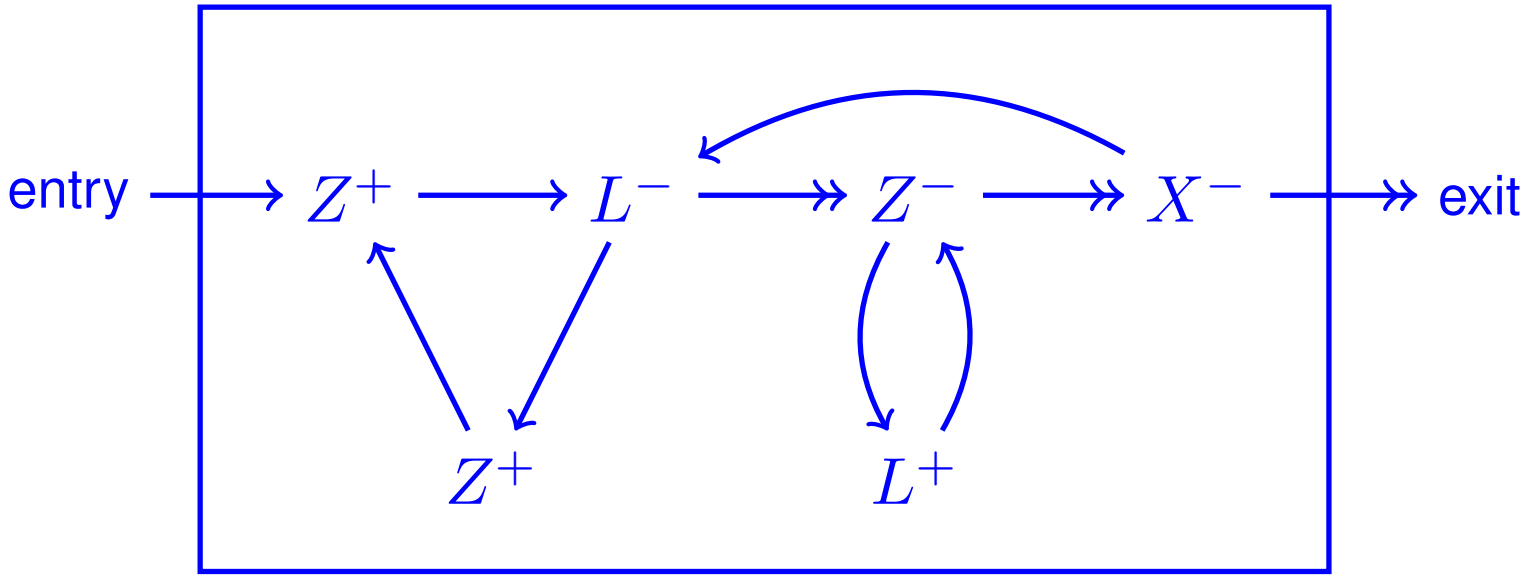
Gadgets: "multiply R_1 by R_2 to R_0"

We can implement "multiply R_1 by R_2 to R_0" by repeated addition:

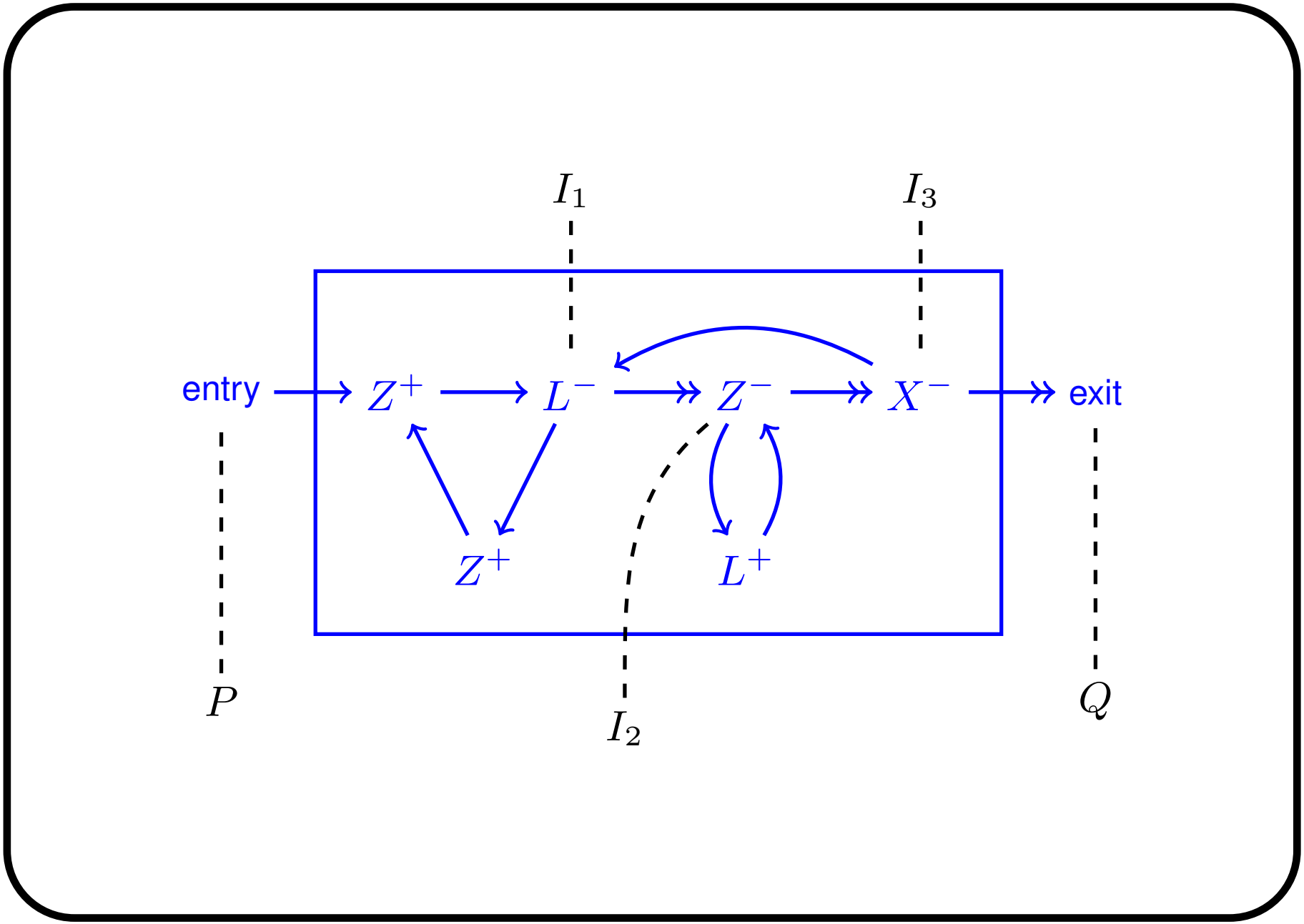


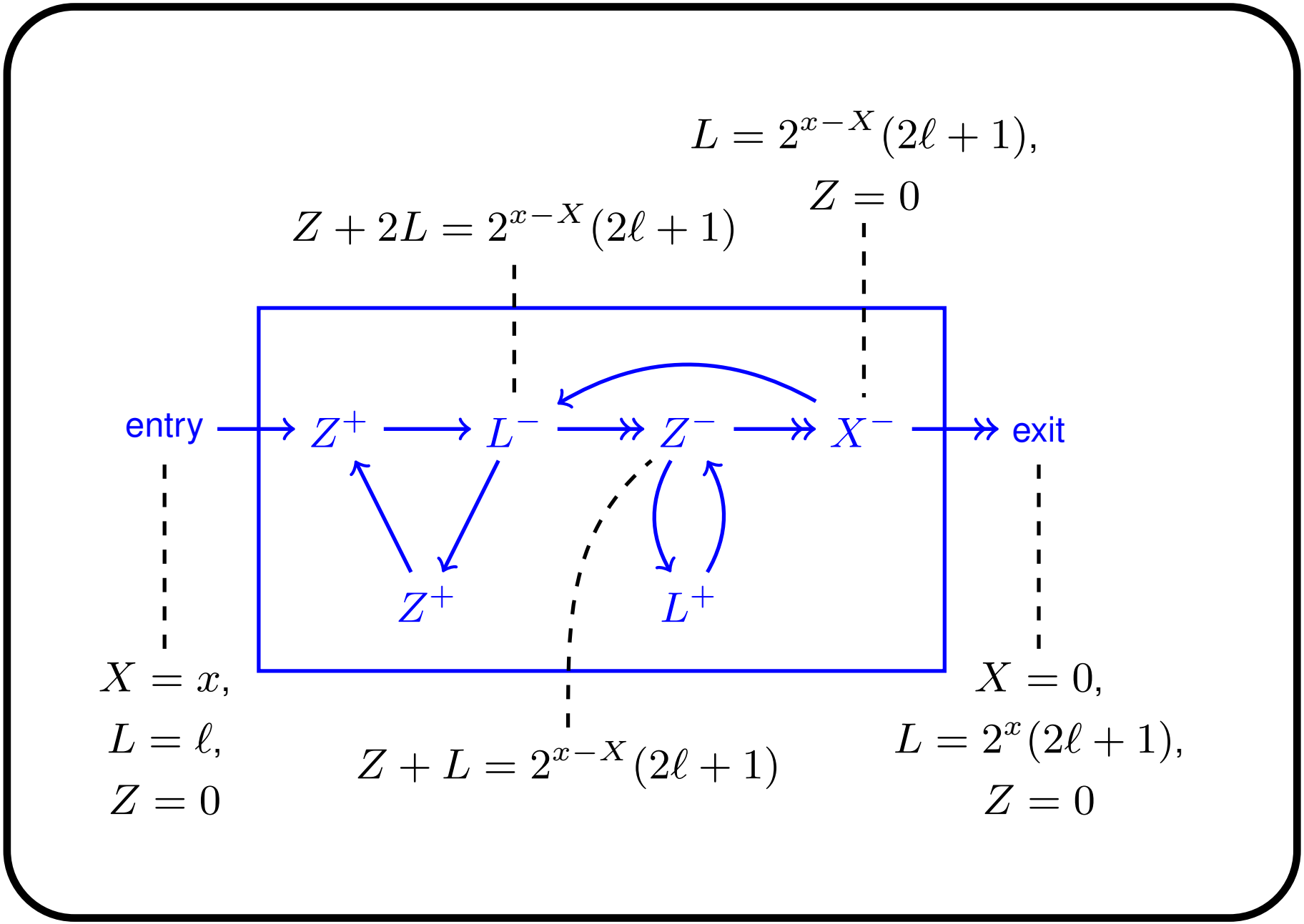
Gadget: "push X to L"

The gadget "push X to L":



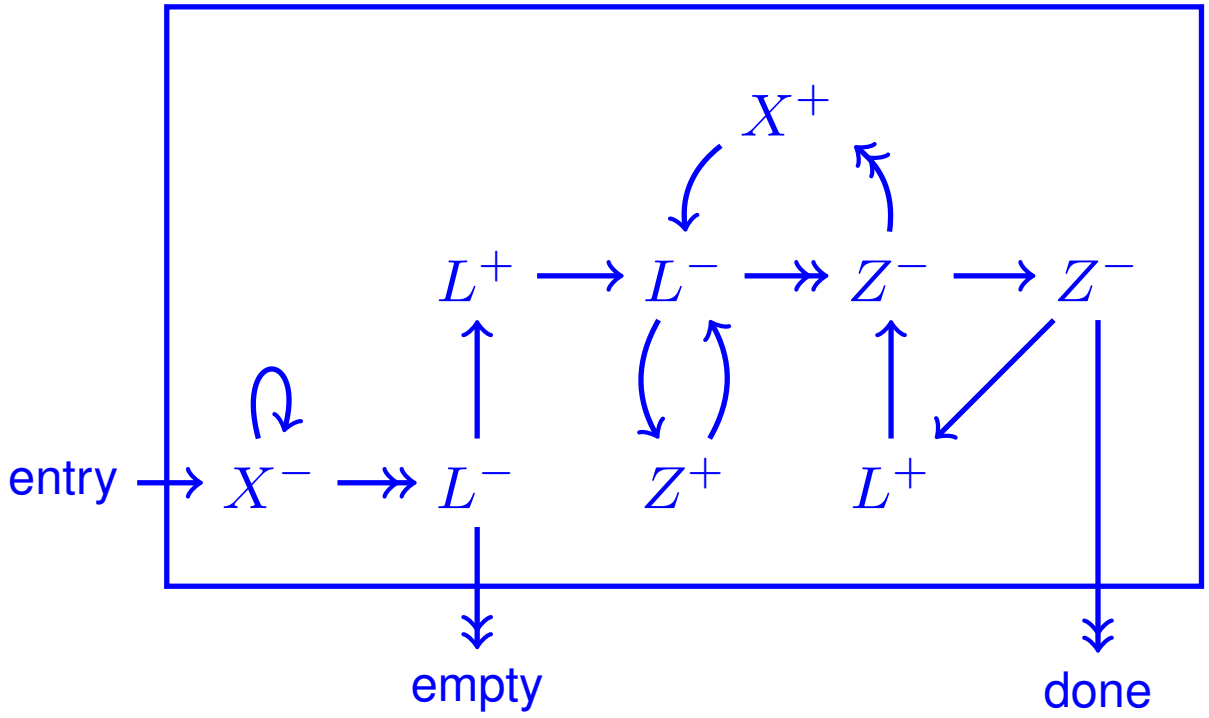
Given input values $X = x$, $L = \ell$ and $Z = 0$, it returns the output values $X = 0$, $L = \langle\!\langle x, \ell \rangle\!\rangle = 2^x(2\ell + 1)$ and $Z = 0$:


I_1
I_3
entry
Z^+
L^-
Z^-
X^-
exit
Z^+
L^+
P
I_2
Q

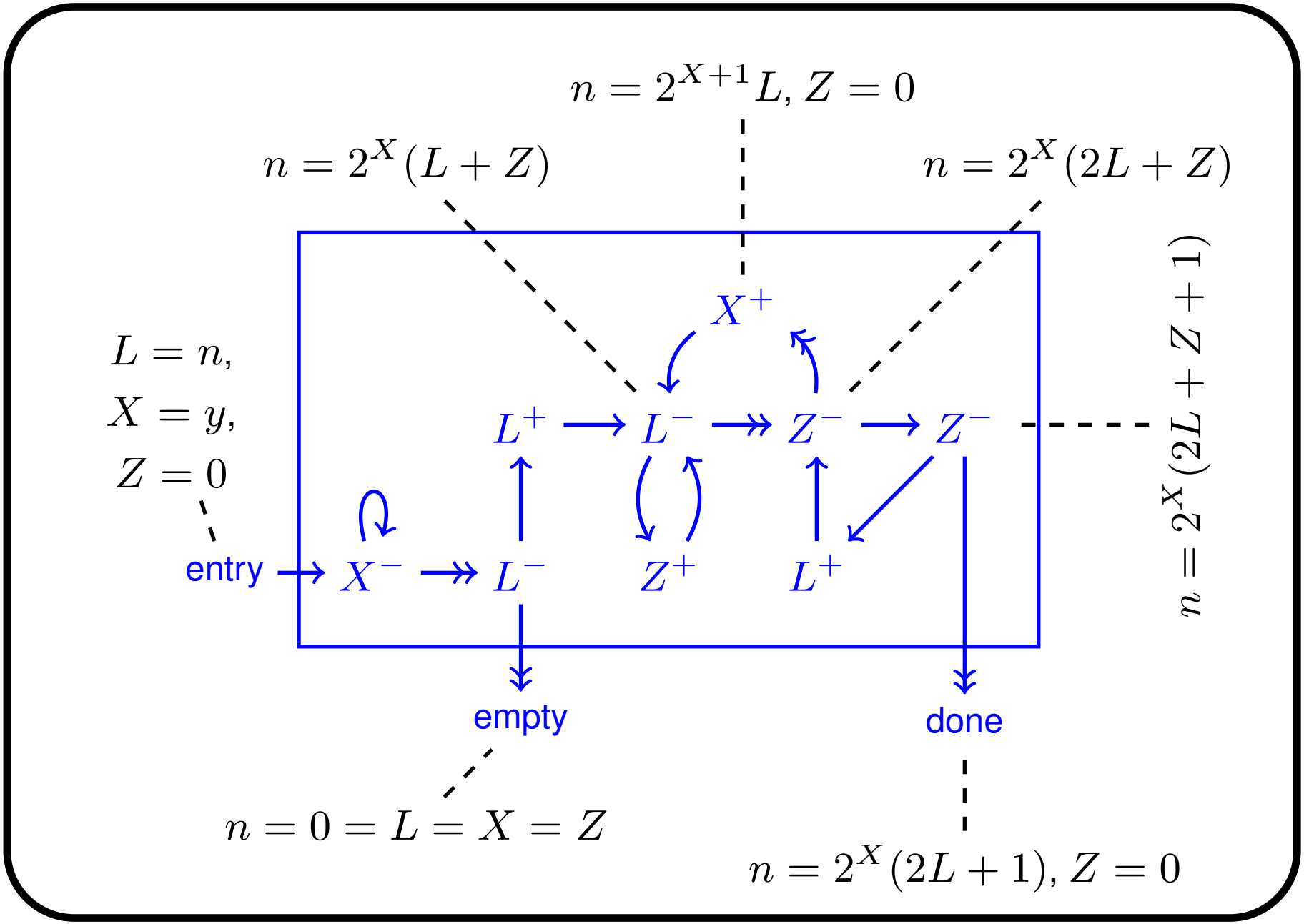

$L = 2^{x-X}(2\ell+1)$,
$Z = 0$
$Z + 2L = 2^{x-X}(2\ell+1)$
entry
Z^+
L^-
Z^-
X^-
exit
Z^+
L^+
$X = x$,
$L = \ell$,
$Z = 0$
$Z + L = 2^{x-X}(2\ell+1)$
$X = 0$,
$L = 2^x(2\ell+1)$,
$Z = 0$

Gadget: "pop L to X"

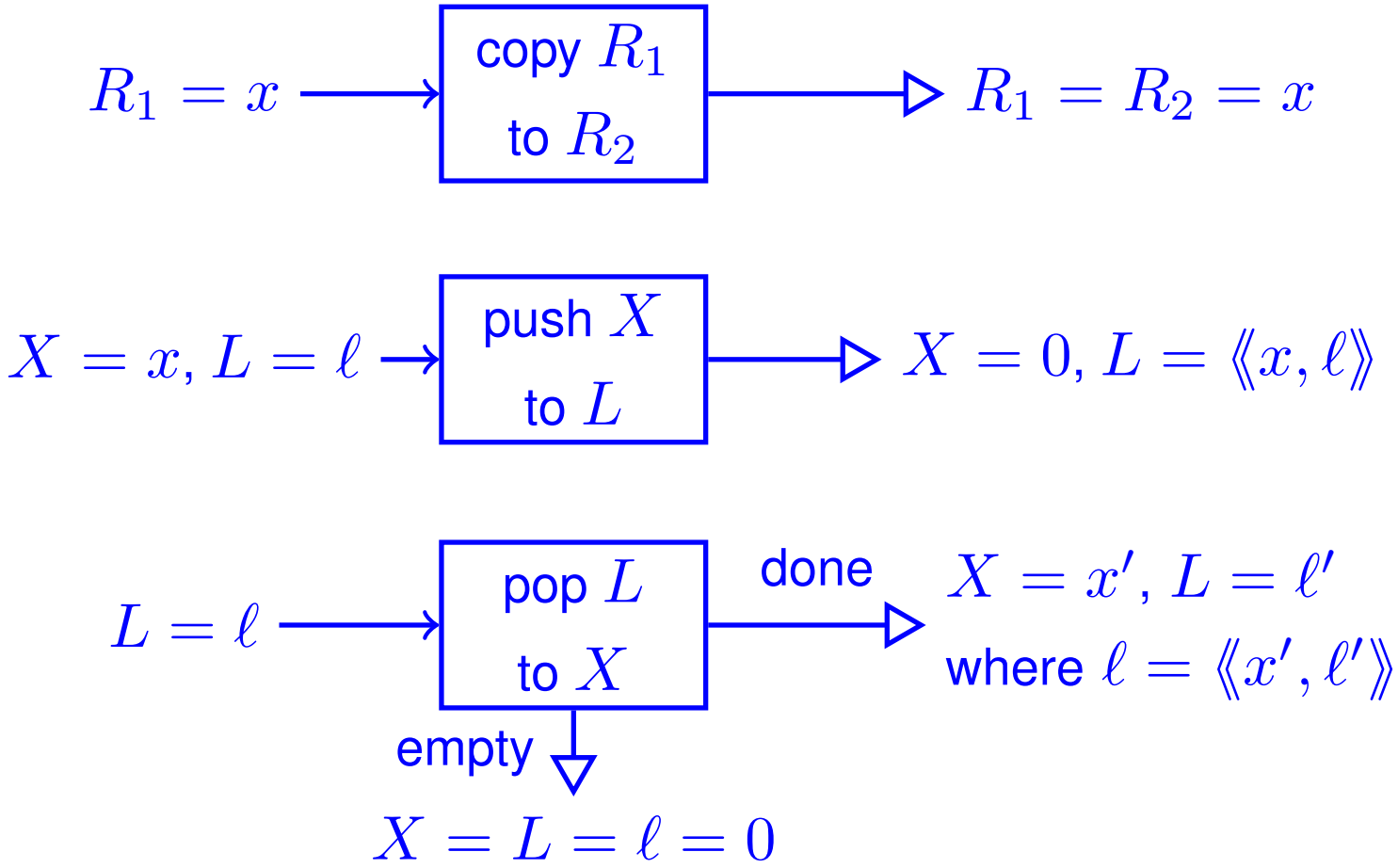
The gadget "pop L to X":



If $L = 0$ then return $X = 0$ and go to "empty". If $L = \langle\langle x, \ell \rangle\rangle = n$ then return $X = x$ and $L = \ell$, and go to "done".


$n = 2^{X+1}L, Z = 0$
$n = 2^X(L + Z)$
$n = 2^X(2L + Z)$
$L = n,$
$X = y,$
$Z = 0$
X^+
L^+
L^-
Z^-
Z^-
$n = 2^X(2L + Z + 1)$
entry
X^-
L^-
Z^+
L^+
empty
done
$n = 0 = L = X = Z$
$n = 2^X(2L + 1), Z = 0$

Gadgets


$R_1 = x$
copy R_1 to R_2
$R_1 = R_2 = x$
$X = x, L = \ell$
push X to L
$X = 0, L = \langle\langle x, \ell \rangle\rangle$
$L = \ell$
pop L to X
done
$X = x', L = \ell'$
where $\ell = \langle\langle x', \ell' \rangle\rangle$
empty
$X = L = \ell = 0$

The Universal Register Machine

The *universal register machine* carries out the following computation, starting with $R_0 = 0$, $R_1 = e$ (code of a program), $R_2 = a$ (code of a list of arguments) and all other registers zeroed:

- decode e as a RM program P
- decode a as a list of register values $a_1, \ldots, a_n$
- carry out the computation of the RM program P starting with $R_0 = 0, R_1 = a_1, \ldots, R_n = a_n$ (and any other registers occurring in P set to 0).

Mnemonics for the registers of U and the role they play in its program:

R_0 result of the simulated RM computation (if any).

$R_1 \equiv P$ Program code of the RM to be simulated

$R_2 \equiv A$ list of RM Arguments (or register contents) of the simulated machine

$R_3 \equiv PC$ Program Counter—label number of the current instruction

$R_4 \equiv N$ label number(s) of the Next instruction(s)—also used to hold code of current instruction

$R_5 \equiv C$ code of the Current instruction body

$R_6 \equiv R$ value of the Register to be used by current instruction

$R_7 \equiv S$ and $R_8 \equiv T$ are auxiliary registers.

R_9**...** other scratch registers.

Overall structure of the URM

1 copy PCth item of list in P to N (halting if $PC >$ length of list); goto **2**

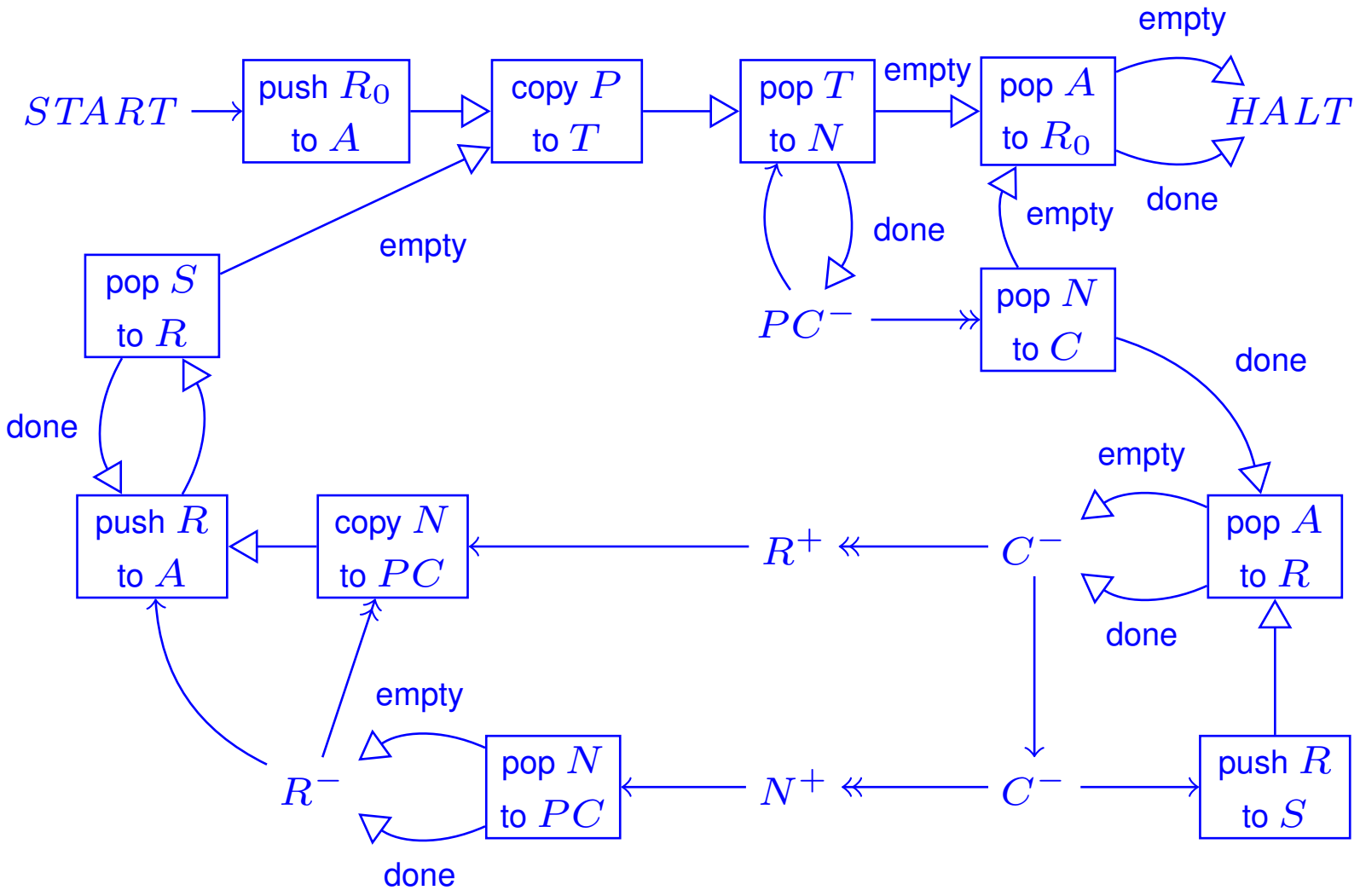
2 if $N = 0$ then halt, else decode N as $\langle\langle y, z \rangle\rangle$; $C ::= y$; $N ::= z$; goto **3**

{at this point either $C = 2i$ is even and current instruction is $R_i^+ \to L_z$, or $C = 2i + 1$ is odd and current instruction is $R_i^- \to L_j, L_k$ where $z = \langle j, k \rangle$}

3 copy ith item of list in A to R; goto **4**

4 execute current instruction on R; update PC to next label; restore register values to A; goto **1**

The Universal Register Machine



Universal Register Machines

Ivan Korec: *Small Universal Register Machines*. Theoretical Computer Science, Volume 168 (1996), pp267–301.

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