Program Analysis (70020)

While Language

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Syntactic Constructs

We use the following syntactic categories:

$$\begin{array}{lcll} a & \in & \textbf{AExp} & \text{arithmetic expressions} \\ b & \in & \textbf{BExp} & \text{boolean expressions} \\ S & \in & \textbf{Stmt} & \text{statements} \end{array}$$

Abstract Syntax of WHILE

The syntax of the language WHILE is given by the following **abstract syntax**:

a

b

S

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b & & \\
S & &
\end{array}
$$

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\end{array}
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S & &
\end{array}
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 & & \mid S_1 ; S_2
\end{array}
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S &::= x := a \\
&\quad \mid \mathbf{skip} \\
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&\quad \mid \mathbf{if}\ b\ \mathbf{then}\ S_1\ \mathbf{else}\ S_2 \\
&\quad \mid \mathbf{while}\ b\ \mathbf{do}\ S
\end{aligned}
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Syntactical Categories

We assume some countable/finite set of variables is given;

$$
\begin{array}{rcll}
x, y, z, \ldots & \in & \textbf{Var} & \text{variables} \\
n, m, \ldots & \in & \textbf{Num} & \text{numerals}
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Numerals (integer constants) will not be further defined and neither will the operators:

$$\begin{array}{rcll} op_a & \in & \mathbf{Op}_a & \text{arithmetic operators, e.g. } +, -, \times, \ldots \\ op_b & \in & \mathbf{Op}_b & \text{boolean operators, e.g. } \wedge, \vee, \ldots \\ op_r & \in & \mathbf{Op}_r & \text{relational operators, e.g. } =, <, \leq, \ldots \end{array}$$

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$$a \quad ::= \quad x$$

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An Example in WHILE

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\textbf{while}\ [y > 1]^3\ \textbf{do}\ (\\
\qquad [\ z := z * y\ ]^4; \\
\qquad [\ y := y - 1\ ]^5); \\
[\ y := 0\ ]^6
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\end{array}
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Note the use of meta-symbols, brackets, to group statements.

Concrete Syntax of WHILE

To avoid using brackets (as meta-symbols) we could also use the **concrete syntax** of the language WHILE as follows:

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$$b \quad ::= \quad true$$

$$S$$

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$$a ::= x \mid n \mid a_1 \ op_a \ a_2$$

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 & & \mid \ S_1 ; S_2 \\
 & & \mid \ \mathbf{if} \ b \ \mathbf{then} \ S_1 \ \mathbf{else} \ S_2 \ \mathbf{fi} \\
 & & \mid \ \mathbf{while} \ b \ \mathbf{do} \ S \ \mathbf{od}
\end{array}
$$

Initial Label

When presenting examples of Data Flow Analyses we will use a number of operations on programs and labels. The first of these is

$$init : \mathbf{Stmt} \to \mathbf{Lab}$$

which returns the initial label of a statement:

$$\begin{array}{rcl} init([\,x := a\,]^{\ell}) & = & \ell \\ init([\,\mathbf{skip}\,]^{\ell}) & = & \ell \\ init(S_1;S_2) & = & init(S_1) \\ init(\mathbf{if}\ [b]^{\ell}\ \mathbf{then}\ S_1\ \mathbf{else}\ S_2) & = & \ell \\ init(\mathbf{while}\ [b]^{\ell}\ \mathbf{do}\ S) & = & \ell \end{array}$$

Final Labels

We will also need a function which returns the set of final labels in a statement; whereas a sequence of statements has a single entry, it may have multiple exits (e.g. in the conditional):

$$\mathit{final} : \textbf{Stmt} \to \mathcal{P}(\textbf{Lab})$$

$$\begin{array}{rcl}
\mathit{final}([\, x := a\,]^\ell) & = & \{\ell\} \\
\mathit{final}([\, \textbf{skip}\,]^\ell) & = & \{\ell\} \\
\mathit{final}(S_1 ; S_2) & = & \mathit{final}(S_2) \\
\mathit{final}(\textbf{if } [b]^\ell \textbf{ then } S_1 \textbf{ else } S_2) & = & \mathit{final}(S_1) \cup \mathit{final}(S_2) \\
\mathit{final}(\textbf{while } [b]^\ell \textbf{ do } S) & = & \{\ell\}
\end{array}$$

The **while**-loop terminates immediately after the test fails.

Elementary Blocks

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- $[\, x := a \,]^{\ell}$, or
- $[\,$ **skip** $]^{\ell}$, as well as
- tests of the form $[b]^{\ell}$.

Blocks

To access the statements or test associated with a label in a program we use the function

$$\mathit{blocks} : \textbf{Stmt} \to \mathcal{P}(\textbf{Block})$$

$$\begin{array}{rcl}
\mathit{blocks}([\, x := a \,]^\ell) & = & \{[\, x := a \,]^\ell\} \\
\mathit{blocks}([\, \textbf{skip} \,]^\ell) & = & \{[\, \textbf{skip} \,]^\ell\} \\
\mathit{blocks}(S_1 ; S_2) & = & \mathit{blocks}(S_1) \cup \mathit{blocks}(S_2) \\
\mathit{blocks}(\textbf{if } [b]^\ell \textbf{ then } S_1 \textbf{ else } S_2) & = & \{[b]^\ell\} \cup \\
 & & \mathit{blocks}(S_1) \cup \mathit{blocks}(S_2) \\
\mathit{blocks}(\textbf{while } [b]^\ell \textbf{ do } S) & = & \{[b]^\ell\} \cup \mathit{blocks}(S)
\end{array}$$

Labels

Then the set of labels occurring in a program is given by

$$\textit{labels} : \textbf{Stmt} \rightarrow \mathcal{P}(\textbf{Lab})$$

where

$$\textit{labels}(S) = \{\ell \mid [B]^{\ell} \in \textit{blocks}(S)\}$$

Clearly $\textit{init}(S) \in \textit{labels}(S)$ and $\textit{final}(S) \subseteq \textit{labels}(S)$.

Flow

$$\mathit{flow} : \textbf{Stmt} \to \mathcal{P}(\textbf{Lab} \times \textbf{Lab})$$

which maps statements to sets of flows:

$$\begin{aligned}
\mathit{flow}([\, x := a \,]^{\ell}) &= \emptyset \\
\mathit{flow}([\, \textbf{skip} \,]^{\ell}) &= \emptyset \\
\mathit{flow}(S_1 ; S_2) &= \mathit{flow}(S_1) \cup \mathit{flow}(S_2) \cup \\
&\quad \{(\ell, \mathit{init}(S_2)) \mid \ell \in \mathit{final}(S_1)\} \\
\mathit{flow}(\textbf{if } [b]^{\ell} \textbf{ then } S_1 \textbf{ else } S_2) &= \mathit{flow}(S_1) \cup \mathit{flow}(S_2) \cup \\
&\quad \{(\ell, \mathit{init}(S_1)), (\ell, \mathit{init}(S_2))\} \\
\mathit{flow}(\textbf{while } [b]^{\ell} \textbf{ do } S) &= \mathit{flow}(S) \cup \{(\ell, \mathit{init}(S))\} \cup \\
&\quad \{(\ell', \ell) \mid \ell' \in \mathit{final}(S)\}
\end{aligned}$$

An Example Flow

Consider the following program, power, computing the x-th power of the number stored in y:

$$
\begin{array}{l}
[\ z := 1\ ]^1; \\
\textbf{while}\ [x > 1]^2\ \textbf{do}\ (\\
\qquad [\ z := z * y\ ]^3; \\
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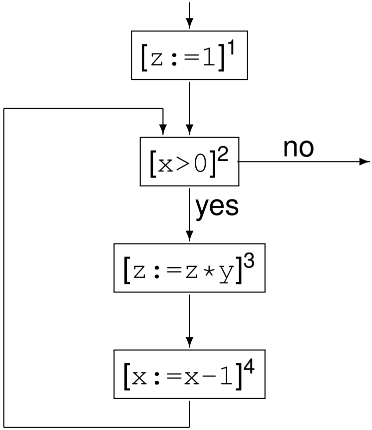
We have *labels*(power) $= \{1, 2, 3, 4\}$, *init*(power) $= 1$, and *final*(power) $= \{2\}$. The function *flow* produces the set:

$$\textit{flow}(\text{power}) = \{(1,2), (2,3), (3,4), (4,2)\}$$

Flow Graph



Forward Analysis

The function *flow* is used in the formulation of *forward analyses*. Clearly $init(S)$ is the (unique) entry node for the flow graph with nodes $labels(S)$ and edges $flow(S)$. Also

$$\begin{array}{rcl} labels(S) & = & \{init(S)\} \cup \\ & & \{\ell \mid (\ell, \ell') \in flow(S)\} \cup \\ & & \{\ell' \mid (\ell, \ell') \in flow(S)\} \end{array}$$

and for composite statements (meaning those not simply of the form $[B]^{\ell}$) the equation remains true when removing the $\{init(S)\}$ component.

Reverse Flow

In order to formulate *backward analyses* we require a function that computes reverse flows:

$$\textit{flow}^R : \textbf{Stmt} \to \mathcal{P}(\textbf{Lab} \times \textbf{Lab})$$

$$\textit{flow}^R(S) = \{(\ell, \ell') \mid (\ell', \ell) \in \textit{flow}(S)\}$$

For the power program, $\textit{flow}^R$ produces

$$\{(2,1),(2,4),(3,2),(4,3)\}$$

Backward Analysis

In case *final*(S) contains just one element that will be the unique entry node for the flow graph with nodes *labels*(S) and edges $\mathit{flow}^R(S)$. Also

$$\begin{aligned}\mathit{labels}(S) &= \mathit{final}(S) \cup \\ &\quad \{\ell \mid (\ell, \ell') \in \mathit{flow}^R(S)\} \cup \\ &\quad \{\ell' \mid (\ell, \ell') \in \mathit{flow}^R(S)\}\end{aligned}$$

Notation

We will use the notation $S_\star$ to represent the program we are analysing (the “top-level" statement)

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- **AExp**$_\star$ to represent the set of *non-trivial* arithmetic subexpressions in $S_\star$ as well as
- **AExp**(a) and **AExp**(b) to refer to the set of non-trivial arithmetic subexpressions of a given arithmetic, respectively boolean, expression.

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Isolated Entries & Exits

Program $S_\star$ has *isolated entries* if:

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Similarly, we shall frequently assume that the program $S_\star$ has *isolated exits*; this means that:

$$\forall \ell_1 \in \mathit{final}(S_\star)\ \forall \ell_2 \in \mathbf{Lab} : (\ell_1, \ell_2) \notin \mathit{flow}(S_\star)$$

Label Consistency

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Clearly, if all blocks in S are uniquely labelled (meaning that each label occurs only once), then S is label consistent.

When S is label consistent the statement or clause "where $[B]^\ell \in \mathit{blocks}(S)$" is unambiguous in defining a partial function from labels to elementary blocks; we shall then say that ℓ labels the block B.