Quantum Computation (CO484)

Quantum Physics and Concepts

Herbert Wiklicky

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Autumn 2017

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9. [Quantum Error Correction]

Practicalities

Two Lecturers

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Teaching $3\frac{1}{2}$ weeks until 30 October

Open-book coursework test 30 October

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Simon Singh: *Code Book*, Forth Estate, 1999.

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Photoelectric Effect – Millikan Experiment

Experimental Setup:

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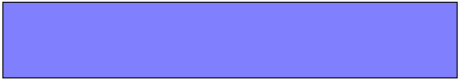

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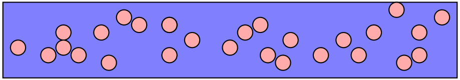

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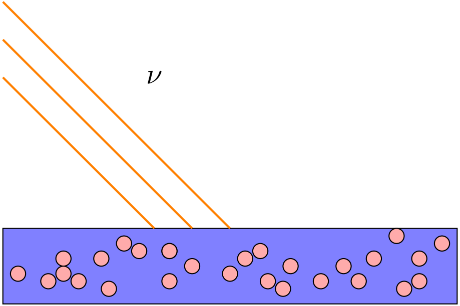

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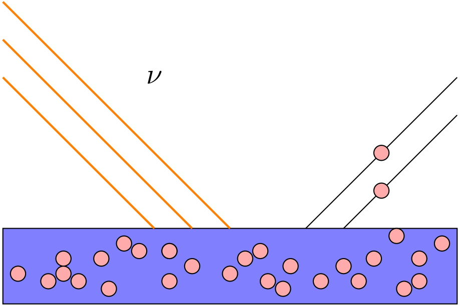

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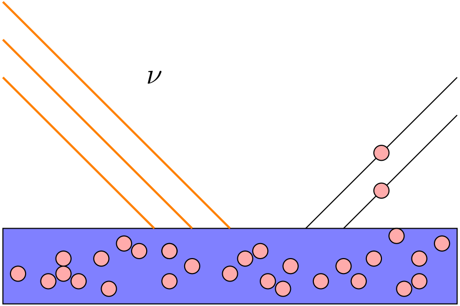

Observed: The velocity, and thus kinetic energy, of the emitted electrons depends not on the intensity of the incoming light but only on its “colour”, i.e. frequency ν.

Radiation Law

Observed relationship:

$$W_k = h\nu - W_e$$

W_k ... Kinetic Energy of Electron

W_e ... Escape Energy of Material

ν ... Frequency of Light

h ... Plank's Constant

$$h = 6.62559 \cdot 10^{-34} Js$$

$$\hbar = \frac{h}{2\pi} = 1.05449 \cdot 10^{-34} Js$$

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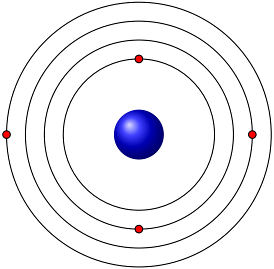

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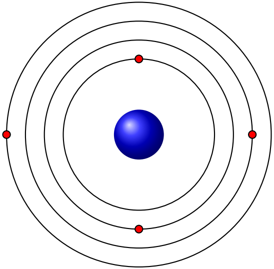

In this way one can also explain the spectral emissions (and absorption) of various elements, e.g. to analyse the material composition of stars (and to make great fireworks).

Quantum Paradoxes and Myths

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7. Whereof one cannot speak, thereof one must be silent.
Ludwig Wittgenstein: *Tractatus Logico-Philosophicus*, 1921

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Each area has its own language which however often applies only to classical entities – for the quantum world we often have simply the wrong vocabulary.

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Related Questions: What is our knowledge of what? How do we obtain this information? What is a description on how the system changes?

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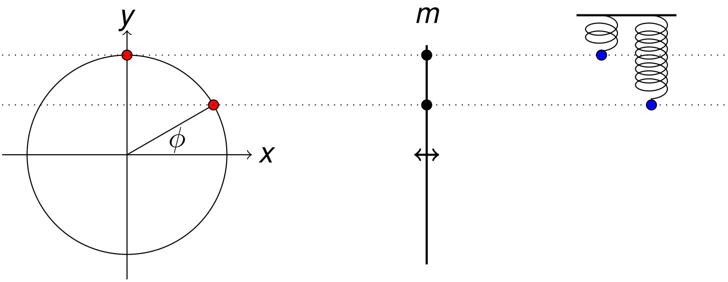

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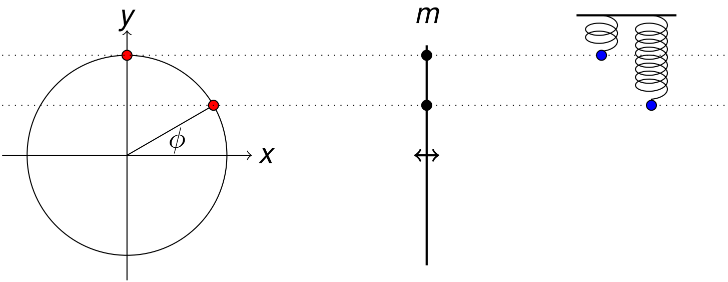

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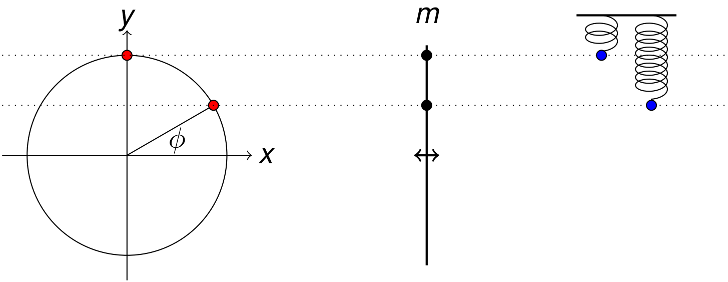

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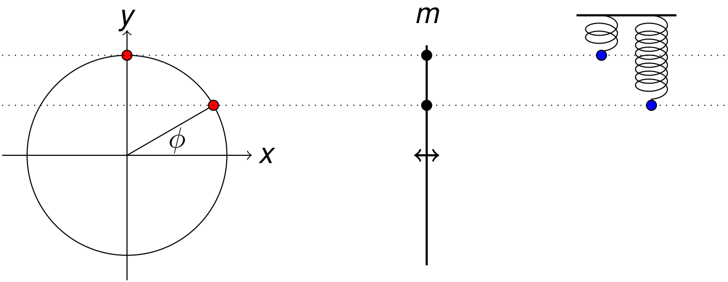

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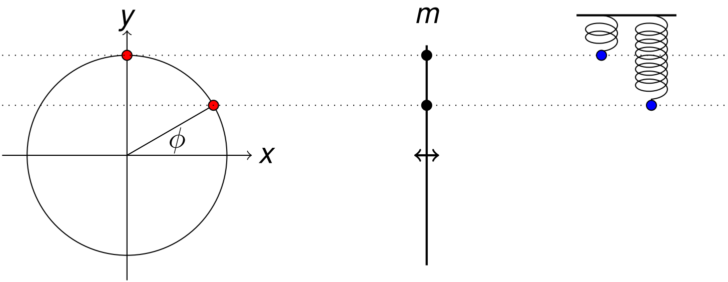

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Dynamics: $(x, y)(t) = (\cos(t), \sin(t))$ or also: $\phi(t) = t$

Postulates for Quantum Mechanics [$*$]

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Key Notions: A quantum systems is (may be) in a certain state, but physicists have to decide which properties they want to observe before a measurement is made (which instrument?).

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- **Probability** to measure (the possible eigenvalue) λ_n if the system is in the state $\vec{\psi} = \sum_i \psi_i \vec{\phi}_i$ is

$$Pr(A = \lambda_n \mid \vec{\psi}) = |\psi_n|^2$$

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There are additional mathematical details in order to deal with “real” quantum physics, e.g. systems an infinite degree of freedom; for quantum computation it is however enough to study finite-dimensional Hilbert spaces.

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Two states can be combined to form a new state $\alpha |x\rangle + \beta |y\rangle$ as long as $|\alpha|^2 + |\beta|^2 = 1$, by **superposition.**

Consequence: We can compute with many inputs in parallel.

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P.A.M. Dirac "invented" the bra-ket notation (most likely inspired by the limitations of old mechanical type-writers); Simply "take the inner product apart" to denote vectors in $\mathcal{H}$:

$$\text{inner product } \langle x|y\rangle \quad = \quad \text{product } \langle x| \cdot |y\rangle$$

For indexed sets of vectors $\{\mathbf{x}_i\}$ (maybe because typographic "typing" was problematic) different notations are used:

$$\mathbf{x}_i = \vec{x}_i = \mathfrak{x}_i = |i\rangle$$

Quantum States and Vectors

Finite quantum states can be described by vectors in $\mathbb{C}^n$, e.g.

$$\vec{\psi} = |\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ or } \langle\phi| = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

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- for enumerating, for example, all eigenvectors of an operator like **A** with $|0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- to enumerate coordinates of one vector, e.g. $\vec{\psi}_1 = 1/\sqrt{2}$, or better perhaps: $|0\rangle_1 = 0$.

Quantum Postulates II – Measurement

- The expected result (average) when measuring observable **A** of a system in state $|x\rangle \in \mathcal{H}$ is given by:

$$\langle A \rangle_x = \langle x | \mathbf{A} | x \rangle = \langle x | | \mathbf{A} x \rangle$$

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then we have: $\mathbf{A} = \sum_i \lambda_i \mathbf{P}_i$ (Spectral Theorem).

Heisenberg's Uncertainty Relation

Theorem

For two observables $\mathbf{A}_1$ *and* $\mathbf{A}_2$ *we have:*

$$(\Delta_{|x\rangle}\mathbf{A}_1)(\Delta_{|x\rangle}\mathbf{A}_2) \geq \frac{1}{2}\left|\left(\langle x|\ [\mathbf{A}_1, \mathbf{A}_2]\ |x\rangle\right)\right|$$

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$$(\Delta_{|x\rangle}\mathbf{A})^2 = \langle x| \ \mathbf{A}^2 \ |x\rangle - \langle x| \ \mathbf{A} \ |x\rangle^2$$

see e.g. Isham: *Quantum Theory*, ICP 1995, Section 7.3.3.

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A standard example of so-called *incomensurable* observables are position $\mathbf{A}_1 = x$ and momentum $\mathbf{A}_2 = p$ (on an infinite-dimensional Hilbert Space $\mathcal{H}$) for which $[x, p] = i\hbar$ and thus:

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In **classical** physics observables always commute, are *comensurable*, i.e. $[\mathbf{A}_1, \mathbf{A}_2] = 0$. In **quantum** physics for most observables $[\mathbf{A}_1, \mathbf{A}_2] \neq 0$, i.e. the observable algebra is typically non-commutative or non-abelian (cf. multiplication of (complex) numbers vs multiplication of matrices).

Quantum Dynamics

- The **dynamics** of a (closed) system is described by the Schrödinger Equation:

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Theorem

For any self-adjoint operator $\mathbf{A}$ *the operator*

$$\exp(i\mathbf{A}) = e^{i\mathbf{A}} = \sum_{n=0}^{\infty} \frac{(i\mathbf{A})^n}{n!}$$

is a unitary operator.

Irreversible vs Reversible

There are a number of immediate consequence of the postulates.

1. The state develops reversibly, i.e. $|x_t\rangle = \mathbf{U}_t\,|x_0\rangle$ for some unitary matrix (operator).
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The mathematical structure has also consequences for any **Quantum Logic**, e.g. De Morgan fails, 'Tertium non datur' is not guaranteed, etc.

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Given a desired computation (dynamics).
What quantum device (e.g. circuit) is needed to obtain this?

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When will (cheap) quantum computers be available? What will be a **killer application** for quantum computation? When will we reach quantum supremacy?