TUTORIAL NOTES 7

12·2005

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dvd Ø3

* Find roots of $-2\sqrt{2} - 2\sqrt{2}\,i$

Solution :—

Rewrite in polar form

$$z^2 = -2\sqrt{2} - 2\sqrt{2}\,i = -2\sqrt{2}\,(1+i)$$

$$= e^{(\pi + 2k\pi)i} \times 2\sqrt{2}\,(1+i)$$

because $(-1) = e^{(\pi + 2k\pi)i}$

$$= e^{i(\pi + 2k\pi)}\, 2\sqrt{2} \times \sqrt{2}\, e^{(\frac{\pi}{4} + 2k\pi)i}$$

because $(1+i) = \sqrt{2}\, e^{(\frac{\pi}{4} + 2k\pi)i}$

$$= 4\, e^{i(\frac{5\pi}{4} + 2k\pi)}$$

So, taking the root gives

$$z = 2\left[e^{i(\frac{5\pi}{4} + 2k\pi)}\right]^{\frac{1}{2}} = 2\, e^{i(\frac{5\pi}{8} + k\pi)}$$

$$= 2\, e^{\frac{5\pi}{8} i}, \quad 2\, e^{\frac{13\pi}{8} i}$$

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* Find z such that $z^5 = -16\sqrt{2}\,(1+i)$

Solution :—

Rewrite in polar form

$$z^5 = -16\sqrt{2}\,(1+i)$$

$$= e^{i(\pi + 2k\pi)} \times 16\sqrt{2} \times \sqrt{2}\, e^{i(\frac{\pi}{4} + 2k\pi)}$$

$$= 32\, e^{i(\frac{5\pi}{4} + 2k\pi)}$$

$$\Rightarrow z = 2\left[e^{i(\frac{5\pi}{4} + 2k\pi)}\right]^{\frac{1}{5}}$$

$$= 2\, e^{i(\frac{\pi}{4} + \frac{2k\pi}{5})}$$

$$= 2\, e^{\frac{\pi}{4} i}, \quad 2\, e^{\frac{13\pi}{20} i}, \quad 2\, e^{\frac{21\pi}{20} i}, \quad 2\, e^{\frac{29\pi}{20} i}, \quad 2\, e^{\frac{37\pi}{20} i}$$