FUNCTION LIMITS & DIFFERENTIATION

25·10·05

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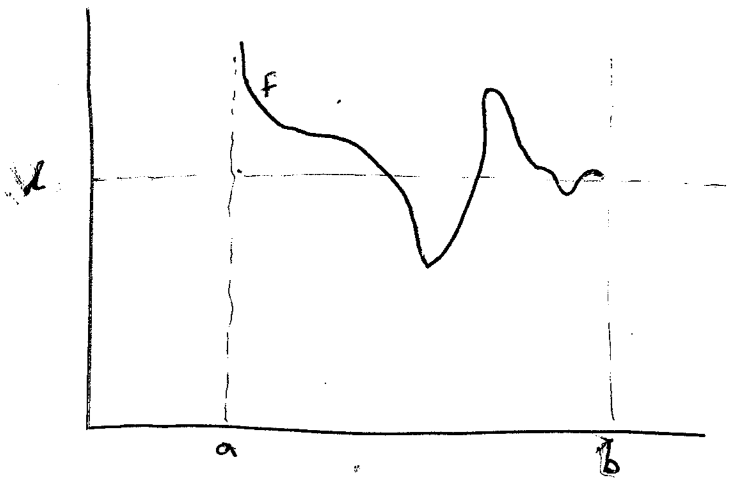
dvd Ø3

LIMITS OF FUNCTIONS

Limit from Left



Function f is defined between a and b — not necessarily at a or b.

We say $f(x)$ tends (or converges) to a limit l as x tends to b *from the left* and write

$$f(x) \to l \text{ as } x \to b^-$$

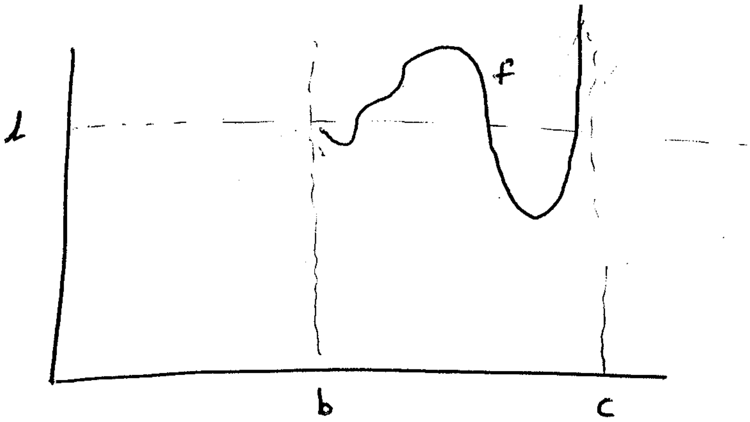
or

$$\lim_{x \to b^-} f(x) = l$$

if

given any $\epsilon > 0$, we can find a $\delta > 0$ such that $|f(x) - l| < \epsilon$ provided that $b - \delta < x < b$

nit from Right



Function f is defined between b and c — not necessarily at b or c

We say $f(x)$ tends to a limit l as x tends to b from the right and write

$$f(x) \to l \quad \text{as} \quad x \to b^+$$

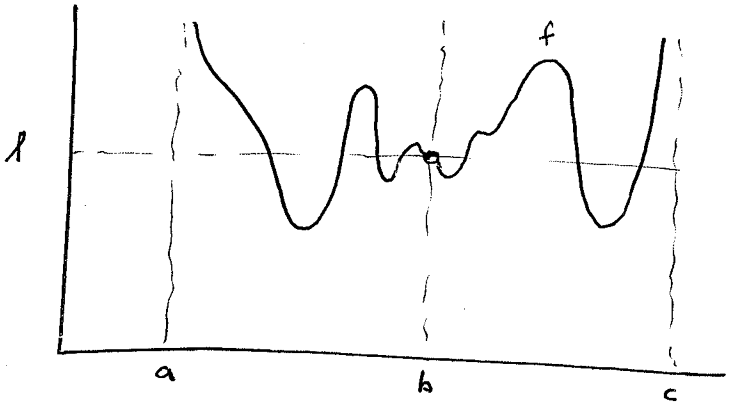
or

$$\lim_{x \to b^+} f(x) = l$$

if

given any $\epsilon > 0$, we can find a $\delta > 0$ such that $|f(x) - l| < \epsilon$ provided that $b < x < b + \delta$

Limit



Function f is defined between a and b and between b and c — not necessarily at a, b or c

We say $f(x)$ tends to a limit l as x tends to b and write

$$f(x) \to l \text{ as } x \to b$$

or

$$\lim_{x \to b} f(x) = l$$

if

given any $\epsilon > 0$, we can find a $\delta > 0$ such that $|f(x) - l| < \epsilon$ provided that $0 < |x - b| < \delta$

i.e. if

$\lim_{x \to b^-} f(x) = l$ <u>AND</u> $\lim_{x \to b^+} f(x) = l$

EXAMPLES

① $\frac{1}{x}$ as $x \to 0$

② $\frac{1}{x}$ as $x \to \infty$

③ $f(x) := \begin{cases} x & \text{if } x \geq 0 \\ x-1 & \text{if } x < 0 \end{cases}$ as $x \to 0$

(4) $\cos x$ as $x \to \infty$

(5) $\dfrac{\cos x}{x}$ as $x \to 0$

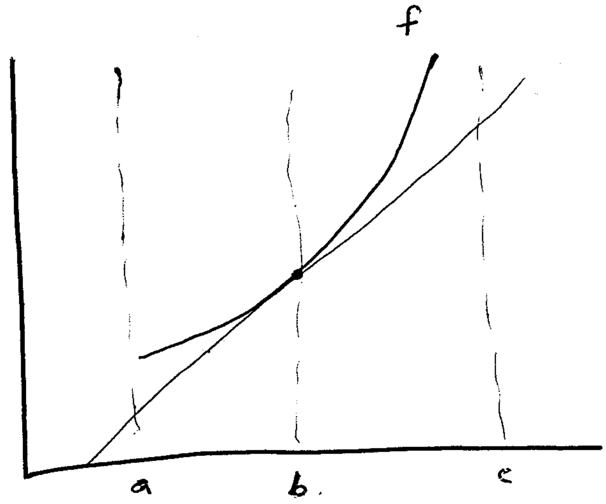
(6) $\dfrac{\cos x}{x}$ as $x \to \infty$

(7) $\left(k - \dfrac{1}{x}\right)$ as $x \to \infty$

(k is some constant $x \in \mathbb{R}$)

DIFFERENTIATION

DERIVATIVE



Function f is defined between a and c — not necessarily at a or c

We say f is differentiable at b if and only if

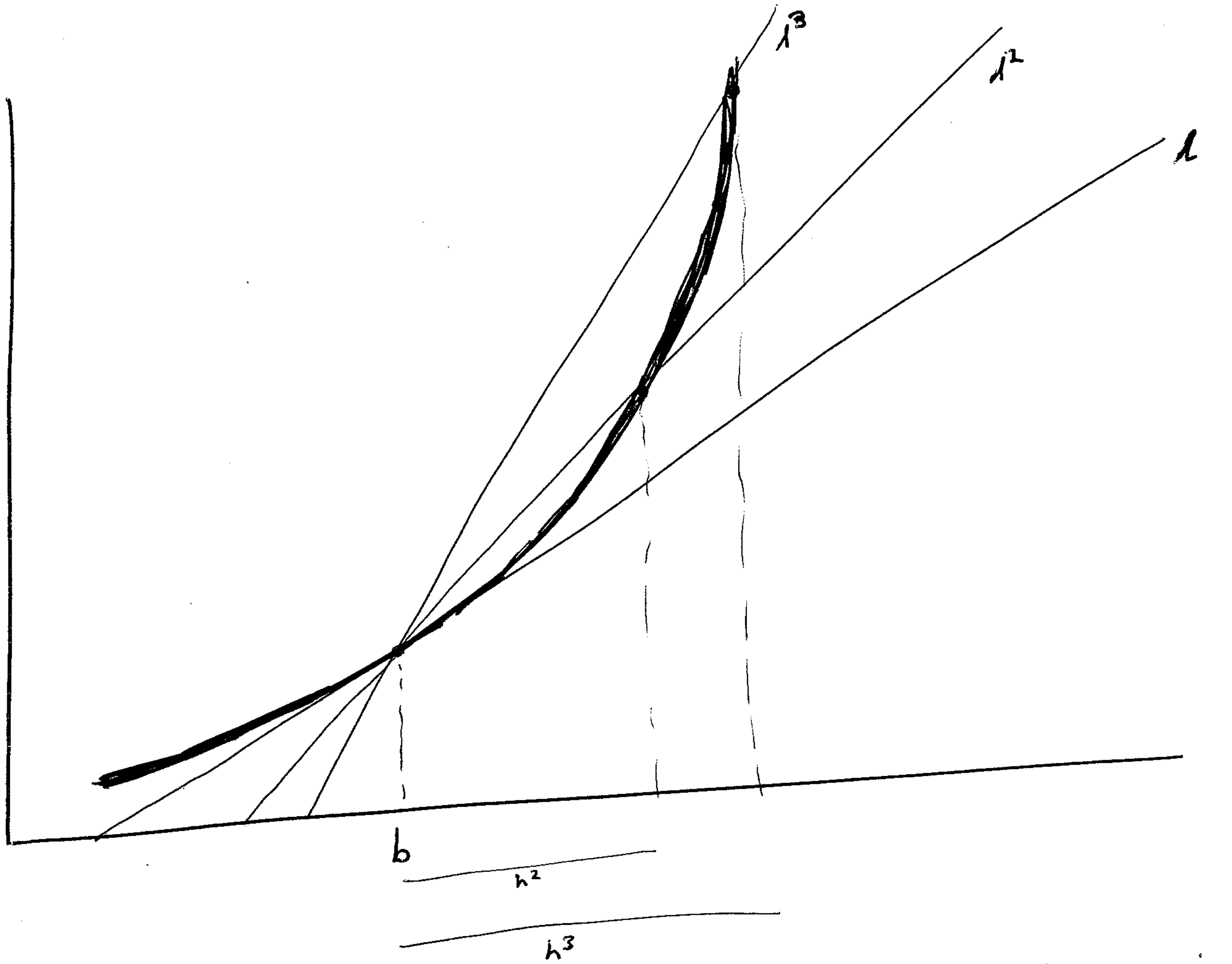
$$\boxed{\lim_{x \to b} \frac{f(x) - f(b)}{x - b} \text{ exists}}$$

If the limit exists we term it the derivative and denote it $f'(b)$. So,

$$\boxed{f'(b) = \lim_{x \to b} \frac{f(x) - f(b)}{x - b}}$$

Or, equivalently,

$$\boxed{f'(b) = \lim_{h \to 0} \frac{f(b+h) - f(b)}{h}}$$



$f'(b)$ is the gradient of f at $x = b$ —— i.e. the gradient of the line l.

Consider the gradient of line l^3:

$$\frac{\Delta y}{\Delta x} = \frac{f(b+h_3) - f(b)}{h_3}$$

Consider the gradient of line l^2

$$\frac{\Delta y}{\Delta x} = \frac{f(b+h_2) - f(b)}{h_3}$$

$$f'(b) = \lim_{h \to 0} \frac{f(b+h) - f(b)}{h}$$

EXAMPLES

① $f(x) := x^2$

find $f'(x)$