TUTORIAL 2

25.10.05

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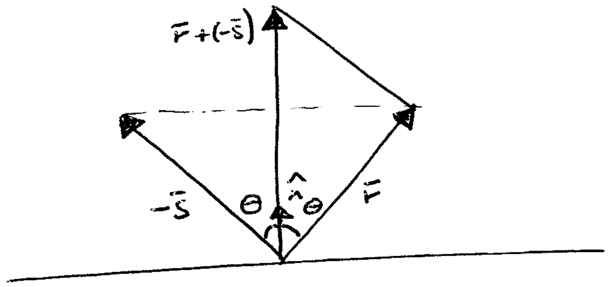
dvd Ø3

REFLECTION

Step 1 State givens.

These will be: $\begin{cases} \bar{s} \\ \hat{n} \end{cases}$

Step 2 Draw picture



Step 3 Derive what we can from the picture

$$(\mathrm{I})\begin{cases} \bar{r}\cdot\hat{n} = r\cos\theta \\ (-\bar{s})\cdot\hat{n} = s\cos\theta \\ \Rightarrow \bar{r}\cdot\hat{n} = -\bar{s}\cdot\hat{n} \end{cases}$$

$$(\mathrm{II})\begin{cases} \bar{r} + (-\bar{s}) = \lambda\hat{n} \\ \Rightarrow (\bar{r} - \bar{s})\cdot\hat{n} = \lambda\hat{n}\cdot\hat{n} \\ \qquad = \lambda \\ \Rightarrow \bar{r}\cdot\hat{n} - \bar{s}\cdot\hat{n} = \lambda \end{cases}$$

The vector $\bar{r}+(-\bar{s})$ is a scalar multiple of the $\perp$ vector $\hat{n}$.

(III) Sub I into II

$\Rightarrow -2\bar{s}\cdot\hat{n} = \lambda$

Solve for λ

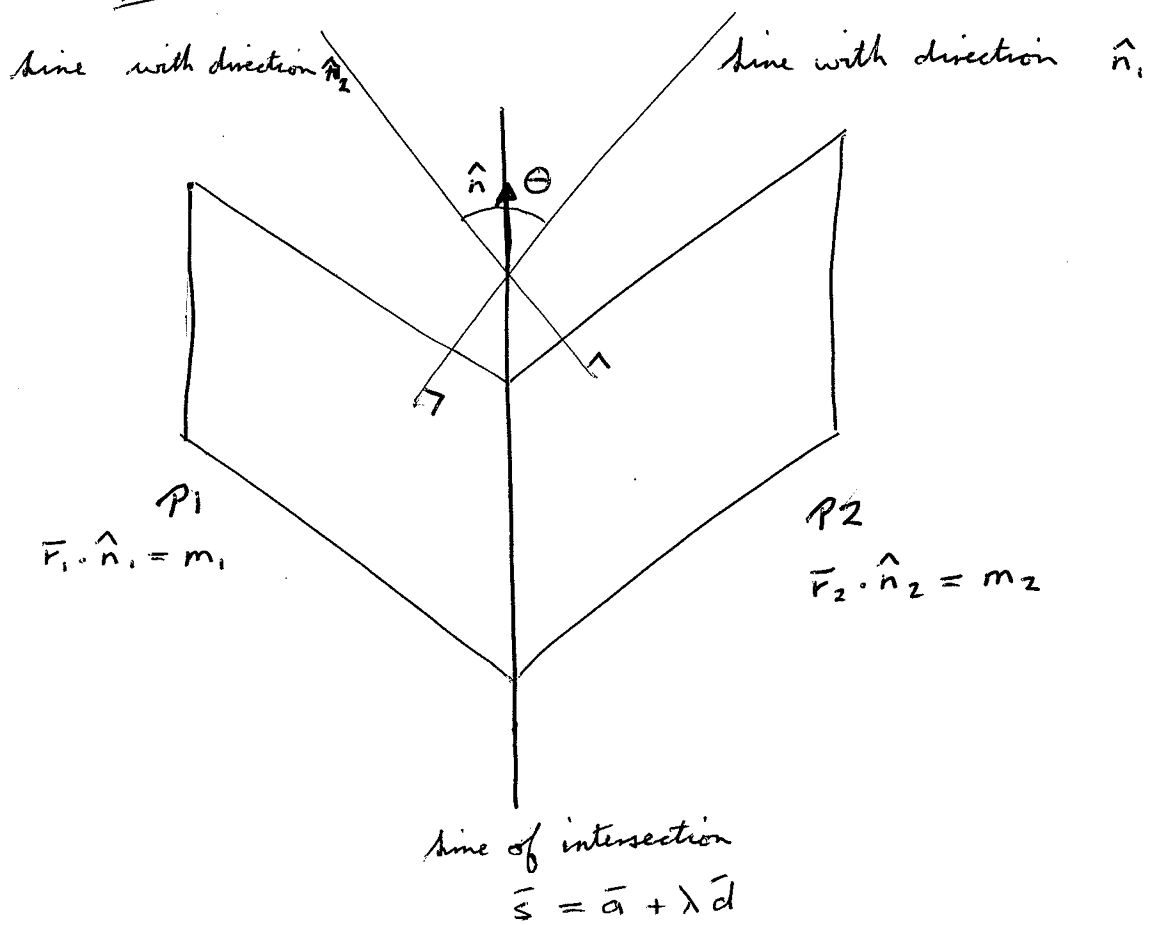
(IV) Rearrange II and sub III into II

[redacted]

$\bar{r} = \bar{s} + \lambda\hat{n}$

Solve for $\bar{r}$

INTERSECTION OF PLANES



Consider $\hat{n}_1 \times \hat{n}_2 := \left((\sin\theta) |\hat{n}_1| |\hat{n}_2| \right) \hat{n}$ —— $*$

$= \sin\theta \, \hat{n}$ —— $**$

where $\hat{n}$ is a unit vector in a direction $\perp$ to $\hat{n}_1$ and $\hat{n}_2$ in a sense of a RH screw turned from $\hat{n}_1$ to $\hat{n}_2$

[NB Check to see that $|\hat{n}_1| = 1$ and $|\hat{n}_2| = 1$. Often aren't in exam questions. If they aren't use $(*)$ rather than $(**)$]

$\boxed{\text{Case 1}}$ $\boxed{\hat{n}_1 \times \hat{n}_2 = 0}$

Iff $P_1 \parallel P_2$ or $P_1 = P_2$, then $\hat{n}_1 \parallel \hat{n}_2$

$\Leftrightarrow \theta = 0$ or π

$\Leftrightarrow \sin\theta = 0$

$\Leftrightarrow \hat{n}_1 \times \hat{n}_2 = 0$

So, $P_1 \parallel P_2$ or $P_1 = P_2$

To check which, we do the following:

Consider P_1:

$$\bar{r}_1 \cdot \hat{n} = m_1$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = m_1$$

$$\Rightarrow x n_1 + y n_2 + z n_3 = m_1 \quad \text{———} \quad ***$$

Choose any values you like for x, y and z which satifies $***$, say $\begin{cases} x = \alpha \\ y = \beta \\ z = \gamma \end{cases}$

See if $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \cdot \hat{n}_2 = m_2$

If not, P_1 and P_2 are parallel, distinct planes.

$\boxed{\text{Case 2}}$ $\boxed{\hat{n}_1 \times \hat{n}_2 \neq 0}$

Recall the defn of $\hat{n}$

This implies that there is a vector, namely $(\hat{n}_1 \times \hat{n}_2)$ (this vector is a scalar multiple of $\hat{n}$) which is $\perp$ to $\hat{n}_1$ and also $\perp$ to $\hat{n}_2$.

Now, all vectors $\perp$ to $\hat{n}_1$ are $\parallel$ to P_1
Also, all vectors $\perp$ to $\hat{n}_2$ are $\parallel$ to P_2

So, the vector given by $(\hat{n}_1 \times \hat{n}_2)$ is $\parallel$ to P_1 and to P_2.

But, then $(\hat{n}_1 \times \hat{n}_2)$ is the direction vector of the line of intersection of P_1 and P_2 (the line of points which lie on both P_1 and P_2).

So, set $\bar{d} := \hat{n}_1 \times n_2$

Any point which lies both on P_1 and P_2 would serve as $\bar{a}$, so to find $\bar{a}$ solve for values a_1, a_2 and a_3 where

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \hat{n}_1 = m_1 \quad \underline{\text{AND}} \quad \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \hat{n}_2 = m_2$$

Having found $\bar{a}$ and $\bar{d}$, we define the line $\bar{s} = \bar{a} + \lambda \bar{d}$

IDENTITY STATEMENTS

Asked to show that $A \equiv B$

Step 1: Expand LHS.

$$
\begin{aligned}
\text{So,} \quad A &\Rightarrow \ldots \\
&\Rightarrow \ldots \\
&\Rightarrow \ldots \\
&\vdots
\end{aligned}
$$

Step 2: Epand RHS

$$
\begin{aligned}
\text{So,} \quad B &\Rightarrow \ldots \\
&\Rightarrow \ldots \\
&\Rightarrow \ldots \\
&\vdots
\end{aligned}
$$

Aim is to reach expansions such that LHS = RHS