**332 Advanced Computer Architecture**

# Unassessed tutorial exercise 2: Assessing a vector processing enhancement

## Background

A vector pipeline processor is an extension to a conventional CPU, which provides registers which can hold an entire vector of floating-point operands (e.g. 64 words). In addition, the machine would have instructions which manipulate vectors. For example, to implement the following loop:

```
for i := 1 to 64 do
  for j := 1 to 64 do
    Y[k,j] := a * X[i,j] + Y[k,j];
```

This could be programmed as follows:

```
R1 := 0
R2 := 0
R3 := k*N

for i := 1 to 64 do
    LV      V2,X(R1)   ; load row of X (R1 is i*N, where N is row length)
    MULTSV  V3,a,V2    ; multiply each element of the row vector by scalar 'a'
    LV      V1,Y(R2)   ; load k'th row of Y, where R2 is k*N)
    ADDV    V4,V3,V1   ; add Y[i] and a*X[i]
    SV      V4,Y(R3)    ; store the vector into array Y

    R1 := R1+N
    R2 := R2+N
end do
```

The classic vector pipeline machine is the Cray 1, and there are many later designs from Cray and others (eg the NEC SX-6 used in the Earth Simulator). The main advantages of vector registers and vector instructions are that pipelined floating point arithmetic can be organised very efficiently, and the maximum throughput of a highly-interleaved memory system can be exploited.

We will study vector processors in more detail shortly. No further details are needed for this exercise.

This exercise concerns Amdahl's Law, which is discussed at length in the textbook:

$$\text{speedup}_{\text{overall}} = \frac{1}{(1 - \text{fractionaccelerated}) + \frac{\text{fractionaccelerated}}{\text{speedup}_{\text{accelerated}}}}$$

**Exercise 1.1. The question**

Assume that we are considering accelerating a machine by adding a vector mode to it. When a computation is run in vector mode, it is 20 times faster than the normal mode of execution. We call the percentage of time that could be spent using vector mode the *percentage of vectorisation*.

1. What percentage of vectorisation is needed to achieve a speedup of 2?
2. What percentage vectorisation is needed to achieve one-half the maximum speedup available from using vector mode?
3. Suppose you have measured the percentage of vectorisation for programs to be 70%. The hardware design group says they can double the the speed of the vector mode with a significant engineering investment. You wonder whether the compiler crew could increase the use of vector mode as another approach to increasing performance. How much of an increase in the percentage of vectorisation (relatitive to current usage) would you need to obtain the same performance gain? Which investment would you recommend?
4. Draw (sketch!) a graph which plots the speedup as a percentage of the computation performed in vector mode. Label the $y$ axis "Net Speedup", and label the $x$ axis "Percent Vectorisation".

*(each part should take only a few minutes; you do not really need a calculator to uncover the basic point of this exercise – back-of-the-envelope estimates are quite adequate).*

## Exercise 1.2. Before and After

Assume that we make an enhancement to a computer that improves some mode of execution by a factor of 10. Enhanced mode is used 50% of the time, measured as a percentage of the execution time when the enhanced mode is in use *(rather than as defined in the notes, where the percentage of the running time* **without** *the enhancement is used).*

1. What is the speedup we have obtained from fast mode?
2. What percentage of the original execution time has been converted to fast mode?

(These exercises are taken from Hennessy and Patterson).

*Paul Kelly, Imperial College, 2007*

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# Notes on the solutions to unassessed exercise 1

## Exercise 1.1

1. For net speedup of 2, we need

$$\frac{1}{(1 - \text{fractionvectorised}) + \text{fractionvectorised}/20} = 2$$

Rearranging Amdahl's Law gives:

$$\text{fractionvectorised} = \frac{\text{speedup}_{\text{overall}} \times \text{speedup}_{\text{accelerated}} - \text{speedup}_{\text{accelerated}}}{\text{speedup}_{\text{overall}} \times \text{speedup}_{\text{accelerated}} - \text{speedup}_{\text{overall}}}$$

So

$$\text{fractionvectorised} = \frac{2 \times 20 - 20}{2 \times 20 - 2} = \frac{20}{38} = 52.6\%$$

2. With the original vector mode design, 70% vectorisation yields a net speedup of $\frac{1}{0.3+0.7/20} = 2.985$. Increasing the vector mode speedup to a factor of 40 would increase this to $\frac{1}{0.3+0.7/40} = 3.15$.

To achieve this net speedup by improving the compiler, we must increase the percentage vectorisation. We have

$$\frac{1}{(1 - \text{fractionvectorised}) + \text{fractionvectorised}/20} = 3.15$$

Rearranging as before, we get

$$\text{fractionvectorised} = \frac{3.15 \times 20 - 20}{3.15 \times 20 - 3.15} = \frac{43}{59.85} = 71.8\%$$

So the compiler crew only have to do a couple of percent better to outperform the proposed hardware development.

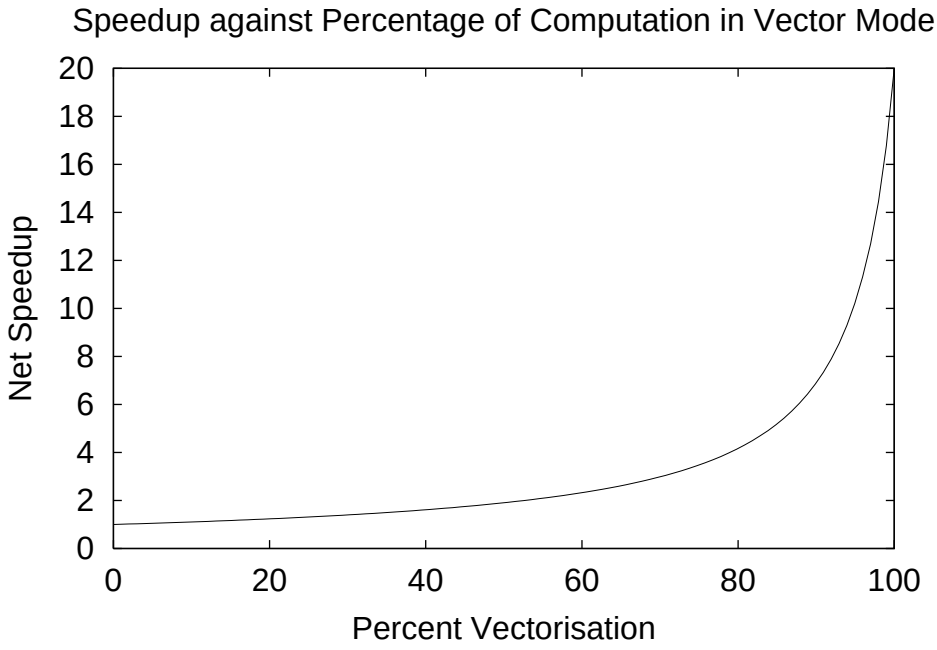
3. To sketch this graph consider a few points, e.g.

- when the percentage vectorised is 0%, net speedup is 1.
- when the percentage vectorised is 100%, net speedup is 20.
- when the percentage vectorised is 50%, net speedup is

$$\frac{1}{0.50 + 0.50/20} = \frac{1}{0.55} = 1.82.$$

- when the percentage vectorised is 75%, net speedup is

$$\frac{1}{0.25 + 0.75/20} = \frac{1}{0.2875} = 3.48.$$




**Exercise 1.2**

1. To compute the speedup obtained from the fast mode we must work out the execution time without the enhancement. We know that the accelerated execution time consisted of two halves: the unaccelerated phase (50%) and the accelerated phase (50%).

   Without the enhancement, the unaccelerated phase would have taken just as long (50%), but the accelerated phase would take 10 times as long, i.e. 500%. So the relative execution time without the enhancement would be $50\% + 500\% = 550\%$.

   Thus the overall speedup is

   $$\frac{\text{executiontime}_{\text{unaccelerated}}}{\text{executiontime}_{\text{accelerated}}} = \frac{550\%}{100\%} = 5.5$$

2. To find the percentage of the original execution time which was accelerated, we plug these figures into Amdahl's Law again:

   $$\begin{aligned} \text{fractionvectorised} &= \frac{\text{speedup}_{\text{overall}} \times \text{speedup}_{\text{accelerated}} - \text{speedup}_{\text{accelerated}}}{\text{speedup}_{\text{overall}} \times \text{speedup}_{\text{accelerated}} - \text{speedup}_{\text{overall}}} \\ &= \frac{5.5 \times 10 - 10}{5.5 \times 10 - 5.5} \\ &= 45/49.5 = 90.90\% \end{aligned}$$

*Paul Kelly, Imperial College, 2007*