

**From Mathematical Logic, to Natural Language, Artificial Intelligence,  
and Human Thinking  
(A short essay in honour of Andrzej Mostowski)**

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**The Stanford/Berkeley-Warsaw Exchange**

I arrived in Warsaw in the summer of 1964, as an exchange student from Stanford University. In addition to two linguistics students, there were two logic students, Peter Hinman from Berkeley and myself. I learned about the Warsaw exchange from Jon Barwise, who had also applied, but was turned down because he was judged to be too young.

Jon and I started our graduate studies together at Stanford in 1963. Jon had just finished his undergraduate studies at Yale University in Connecticut, in three years instead of the usual four. I had just finished my undergraduate studies at the University of Bridgeport, also in Connecticut, but in five years, instead of four, after starting at the University of Chicago and losing a year between Chicago and Bridgeport.

The academic year in Warsaw did not count towards my academic studies at Stanford. So I was free to take a relaxed approach towards my studies in Warsaw. I took advantage of this freedom, to learn Polish, to meet and spend time with my Polish relations, and to meet and marry my Polish wife.

Although both of my parents were of Polish origin, they were both born in Bridgeport in the Connecticut. However my mother returned to Poland with her parents when she was still a child, and both of her brothers were born there. She returned to the United States at the age of seventeen, leaving her family behind.

I attended a primary school, Saint Michael's, attached to a Polish parish. In the first grade, we learned the catechism in Polish: *Dlaczego Pan Bog nas stworzył?* (Why did God make us?) And we learned Polish *koledy* (Christmas carols). But our Polish lessons were discontinued after the first year, and I didn't learn any more Polish in school afterwards.

Professor Mostowski and his wife invited Peter Hinman and me to his house for tea and cake, shortly after our arrival. They made us feel very much at home. Later I attended his seminar on set theory, where the main topics were Paul Cohen's recent proof of the independence of the continuum hypothesis and large cardinal numbers. Witold Marek was one of the more prominent members of the seminar series and one of the more enthusiastic inventors (or discoverers?) of ever larger cardinals. I also attended seminars

by Professors Rasiowa and Grzegorzczuk. It was at Professor Rasiowa's seminar that I met my future wife, Danusia.

### **Logic and natural language**

Although I studied mathematical logic at Stanford and Warsaw, my interest in logic was not primarily mathematical. I was more interested in the use of logic to improve ordinary human reasoning. My interest in this was awakened by a short exposure to propositional logic in the compulsory, first year, discrete mathematics course at the University of Chicago.

My one year and two months at the University of Chicago were traumatic for me for a number of reasons. Among these was the fact that, at the beginning of the first year, I failed the English placement examination and had to take a non-credit, remedial course in English composition. I finished the year with good A's in all my subjects, except for English, in which I received a poor D. However, I was determined to understand what was wrong with my English and how I could improve it. Eventually, after much independent reading and study, I convinced myself that my problem was that I was treating English as means of self-expression rather than as a medium of communication.

I began to realise that self-expression is a solitary activity, in which a single person attempts to put into words thoughts that are in its own mind. Communication, on the other hand, is a social activity, in which the person attempts to put into words thoughts that it wants to be in another person's mind.

I learned that to communicate effectively, you need to express yourself as clearly and as simply as possible. You need to express yourself clearly, so that your readers (or listeners) understand what you intend and do not understand something else. And you need to express yourself simply, so that your readers do not expend unnecessary effort to extract a useful form of your intended meaning. I eventually convinced myself that both of these characteristics of effective communication have a logical interpretation.

The logic of clarity includes avoiding ambiguity. Pronouns, for example, should have unambiguous referents. Not: "Krysia attended Marysia's logic course. She loved the course." But: "Krysia attended Marysia's logic course. Krysia loved the course." Or: "Krysia attended Marysia's logic course. Marysia loved the course."

Even more obviously, the scope of connectives should be unambiguous. Not: "Krysia will teach logic and Marysia will teach logic or Marysia will teach computing." But: "Krysia and Marysia will teach logic, or Marysia will teach computing." Or: "Krysia will teach logic, and Marysia will teach logic or computing." Similarly, not: "Marysia only teaches logic." (She doesn't sleep, doesn't eat, etc.) But: "Marysia teaches only logic."

The logic of simplicity, on the other hand, is the choice, from among logically equivalent sentences, sentences which are easier for the reader to understand and to use for other purposes. For example, not: "If Krysia teaches logic, then Marysia teaches computing if

Basia teaches English.” But: “If Krysia teaches logic and Basia teaches English, then Marysia teaches computing.” And not “I will attend the logic course or you will not teach it”. But I will attend the logic course, if you teach it.”

Of course, clarity and simplicity are not enough. Language also needs to be coherent. For example: “Krysia teaches if Marysia studies. Marysia studies if Basia pays. Basia pays.” But not the logically equivalent: “Basia pays. Krysia teaches if Marysia studies. Marysia studies if Basia pays.”

I was given the opportunity to try out my emerging understanding of these ideas during my stay in Warsaw. Maciej Mączyński, who had visited Stanford the previous year on the same exchange, and who was translating the book “Set Theory with an Introduction to Descriptive Set Theory” by Kuratowski and Mostowski [8], asked me to help him with the translation. We worked out a routine, by means of which Maciej would write the first draft of the translation and I would work on the second draft, improving his English. One day, Maciej informed me that Professor Mostowski wanted to meet me to discuss a problem with the translation.

The problem, it turned out, is how to distinguish between restrictive and non-restrictive relative clauses in English. For example, “*which*” in the sentence “Let  $P$  be a non-empty subset of  $A$  *which* contains no first element.” Introduces a restrictive relative clause. Professor Mostowski maintained that the sentence should be “Let  $P$  be a non-empty subset of  $A$  *that* contains no first element.” He insisted that the correct relative pronoun for restrictive clauses, as in this sentence, is “*that*”, and that the correct pronoun for non-restrictive clauses, as in the sentence “The logic course, *which* was the most enjoyable course Krysia had ever taken, was taught by Marysia.” is “*which*”.

Professor Mostowski wanted to make sure that I understood the difference between restrictive and non-restrictive clauses: namely that restrictive clauses add extra conditions to the noun phrase which they modify, whereas non-restrictive clauses add extra information, which can be expressed equivalently in a separate sentence. Independently of the choice of relative pronoun”, non-restrictive clauses should be set off from the main clause by commas, but restrictive clauses should not. Moreover, “*which*”, rather than “*that*” should be used for non-restrictive clauses.

When I had satisfied him that I understood the difference between the two kinds of relative clauses and that I knew the rule about commas, he accepted my assurances that in the case of restrictive clauses “*that*” and “*which*” are interchangeable. What I didn’t know then, but do know now, is that he was right about the distinction between “*that*” and “*which*” in traditional English, but that I was also right that by the 1970s the traditional distinction had largely fallen out of use.

### **Back in the USA and Beyond**

I left Poland with a better knowledge of both Polish and English and with my newly wed Polish wife. But I was disillusioned, not so much with mathematical logic, but with

mathematics more generally; and when I returned to Stanford I couldn't settle down to my studies. I left Stanford with enough credits for a Master's degree, and got a job teaching mathematics at the Inter-American University in San Juan, Puerto Rico.

I soon realised, however, that I wasn't going to accomplish very much without a Ph.D., and I left Puerto Rico after only a year, to study at the University of Edinburgh. My Ph.D., which I completed in 1970, was in the field of automated theorem-proving. This led to my work on logic programming [3], in which a certain kind of resolution theorem-prover treats logical implications as goal-reduction procedures. I collaborated with Alain Colmerauer in Marseille, who developed the programming language, Prolog, based on this idea. My book "Logic for Problem Solving" [4], published in 1979, aimed to be "an introduction to logic, the theory of problem-solving, and computer programming", all in one.

I left Edinburgh in 1975, joining the Department of Computing at Imperial College in London, working in the area of logic for artificial intelligence. One of the main topics of my research, developed with Marek Sergot, was legal reasoning. We applied logic programming and its extensions to the representation of the 1981 British Nationality Act [9]. Another topic, also developed with Marek, was the event calculus [5], a logic programming representation of causal reasoning, which has been used for such applications as the formalization of tense and aspect in natural language.

I also worked on the development of abductive logic programming [2], which combines predicates defined by logic programs with undefined (or abducible) predicates constrained by integrity constraints. This work gave rise to two further developments. One was the development of an argumentation theory, which led to the demonstration that most logics for default reasoning can be regarded as special cases of assumption-based argumentation [1].

The other development was the embedding of abductive logic programming as the thinking component of intelligent agent interacting with a changing environment. Working mainly with Fariba Sadri, we developed an agent model [6] in which beliefs are represented by logic programs and goals are represented by integrity constraints. The goals can include maintenance goals, achievement goals, prohibitions, and condition-action rules. Observations and actions are represented by abducible predicates.

I took early retirement in 1999, and became an Emeritus Professor and Senior Research Fellow, so that I could change the focus of my research and return my attention to the original concern that attracted me to logic in the first place, to improve the quality of human thinking [7]. I am pleased to acknowledge that my studies of mathematical logic and my work in computational logic have been a useful detour, helping me to develop the tools and techniques needed for this purpose.

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