Lemma 1

(1) $P, \Gamma \vdash e : t$

(2) $P, h \vdash \sigma$

then

(3) $P, h \vdash [e]_\sigma : t$

Proof by structural induction on (1). Let us consider the last rule applied in (1).

1st Case ST-NULL; use RT-NULL

2nd Case ST-VOID-VAL; use RT-VOID-VAL

3rd Case ST-This.x. then

3a Case $e = this$. then $[e]_\sigma = \sigma(this)$. Apply

Def 1.

3b Case $e = x$ similar

4th Case ST-NEW; apply RT-NEW

5th Case ST-INV then

(4) $e = e_1.m(e_2)$

Therefore, there exist c, t' so that

(5) $P, \Gamma \vdash e_1 : c$

(6) $P, \Gamma \vdash e_2 : t$

(7) $\mathcal{M}(P, c, m) = t \ m(t')$

Apply I.H. on (5) and obtain

(8) $P, h \vdash [e_1]_\sigma : c$

Apply I.H on 6, get

(9) $P, h \vdash [e_2]_\sigma : t$

Apply RT-INV on 8, 9 and 7 and get 3.

<u>6th Case</u> ST-SEQ similar to 5th case

<u>7th Case</u> ST-Sub-CLASS, therefore, $\exists c'$, so that

(4) $P, \Gamma \vdash e : c'$

(5) $P \vdash_{l_4} (c') = t$

Apply I.H on 4 and get

(6) $P, h \vdash [e]_\sigma : c'$

Apply RT-Sub-Class on (6) and obtain 3.