Lemma 3 If

(1) $P \vdash h$

(2) $\vdash P$

(3) $P, h \vdash e : t$

(4) $e, h \leadsto e', h'$

Then

(5) $P \vdash h'$

(6) $P, h' \vdash e' : t$

Proof by structural induction on 4. Case analysis over last rule applied

1st Case NEW. Therefore

(7) $e = \text{new } C$

(8) $e' = \iota, \quad \iota$ new in h

(9) $h' = h[\iota \mapsto C \,\| m_1 \mapsto \emptyset. \ \ m_n \mapsto \emptyset \,]]$

(8) $\{m_1 .. m_n\} = \mathcal{M}^a(P, C)$

Because of (7) and (3) we have that

(9) $P \vdash C \leq t$

Therefore

6 is established from 7, 9, RT-ADDR, RT-SUBCLASS

From (7) and WF heap

5 is established.

2nd Case ASYNC

Therefore, we obtain

(7) $e' = \text{void Val}$

(8) $h(\iota) = [\![c \,\|\, qs]\!]$

(9) $\mathcal{M}(P, c, m) = \text{async } m(t')$

(10) $h' = h[\iota \mapsto [\![c \,\|\, qs[m \mapsto \{v\} \cup qs(m)]]\!]]$

(11) $e = \iota.m(v)$

Consider (3). The last rule applied could be INV or SUB

2.1st Case Last rule in (3) was INV.

(12) $P, h \vdash \iota : c''$

(13) $\mathcal{M}(P, c''', m) = t'' \; m(t''')$

(14) $P, h \vdash v : t''''$

From (12) and (2) we obtain

(15) $P \vdash c \leq c''$

From (13) and (9) and (1) and Aux we obtain:

(16) $t'' = \text{async}$ or $t'' = \text{void}$, $t' = t'''$

From (14) and (16) and (1) we obtain

(17) $P, h' \vdash \iota : c'$

and (17) together with (11) and (1) and 5 is established.

Also 6 is established from 7, 16 and RT.SUB.Asyn

2.2 Case Last rule was SUB. Can only be the case were reflexive, so back to 2.1 Case

3rd Case JOIN Therefore,

(7) $e = \iota.m(v)$

(8) $h(\iota) = [\![c \,\|\, qs]\!]$

(9) $(\{m_1 .. m_n\}, e'') \in Schs(P, c, m)$

(10) $qs(m_i) = \{v_i\} \cup q_i \quad \forall i \in 1..n$

(11) $e' = e''[v_1/m_1\_x, \ldots v_n/m_n\_x]$

(12) $h' = h[\iota \mapsto [\![c \,\|\, qs[m_1 \mapsto q_1, .. m_n \mapsto q_n]]\!]]$

Consider (13). Therefore

(13) $P, h \vdash \iota : c'$

(14) $\mathcal{M}(P, c', m) = t' \; m(t'')$

(15) $P, h \vdash v : t''$ (16) $P \vdash t' \leq t$

Taking (7) and (13) we obtain

(17) $P \vdash c \leq c'$

Taking (9) and (1) we obtain

(18) $\mathcal{M}(P, c, m_i) = async \; m_i(t_i)$

(19) $\mathcal{M}(P, c, m) = t''' \; m(t'''')$

(20) $P, (m_1\_x \mapsto t_1, .. m_n\_x \mapsto t_n, x \mapsto t'''', this \mapsto c) \vdash e'' : t'''$

Also, because of 19, 14, 17 and 1, we obtain

(21) $t' = t'''$ or $t', t''' \in \{void, async\}$, $t'''' = t''$

From 8, 18, 10 and 5 we obtain

(21) $P, h \vdash v_i : t_i$

Therefore, from (21), (20), (12), (11) lemma 1, we obtain

(22) $P, h \vdash e' : t'''$

And using (22) and lemma 2, we obtain

(23) $P, h' \vdash e' : t'''$

Thus,

5 is established from 1 and 18.

6 is established from 23, 21 and 16

<u>4th Case</u> Ex, or Ex-Prop through application of RT-Ex.

<u>5th Case</u> SEQ, through application of RT-SEQ.

<u>6th Case</u> CNTX. By case analysis over the structure of E[.].