Lemma 5 If

(1) $P_{+F}, h_{+F} \vdash e_{+F} \sim e$ at κ

(2) $P_{+F} \vdash h_{+F} \sim h$ at $\kappa_1 .. \kappa_n, \kappa$

(3) $e, h \rightsquigarrow e', h'$

Then, there exist $e'_{+F}, h'_{+F}, \kappa'$ with

(4) $e_{+F}, h_{+F} \rightsquigarrow e'_{+F}, h'_{+F}$ or

$e_{+F} = e'_{+F}, h_{+F} = h'_{+F}$

(5) $P_{+F}, h'_{+F} \vdash e'_{+F} \sim e'$ at κ'

(6) $P_{+F} \vdash h'_{+F} \sim h'$ at $\kappa_1 .. \kappa_n, \kappa'$

Proof By structural induction over (3.)

Base Case The depth of (1) is 1. We proceed by case analysis over the last rule applied in (1).

1st Case CONG-BASIC

Therefore

(7) $c = C(e_{+F})$ $\kappa = 0$.

Proceed with case analysis over c_{+F}.

1.1 Case (8) $c_{+F} =$ new C

Therefore, with (7),

(9) $e =$ new c, where null)

Therefore, by construction, there exists $a, \ldots$ that

(10) L free in h,

(11) $h' = h[L \mapsto [\![C \parallel qs :: qs']\!]]$,

(12) $dom(qs) = M^a(P_{+F}, C)$,
$\forall z \in dom(qs): \quad qs(z) = \emptyset$

(13) $dom(qs') = \mathcal{F}_S(P_{+F}, C)$
$\forall f \in dom(qs'): \quad qs'(f) = \emptyset$

(14) $e' = L.\,init(nulls)$

Take $\kappa' = L$ except $\mathcal{F}_S(P_{+F}, S)$. Then, with $e'_{+F} = e_{+F}$, $h'_{+F} = h$

4.5 is established.

Furthermore, because of (10), we have that $\forall j \; 1 \le j \le n \quad \kappa_j = L' \ldots \Rightarrow L' \neq L$. Therefore,

(15) $\vdash \kappa_1 \ldots \kappa_n, \kappa'$

Furthermore, because of (10), applying CONC-obj-1 we obtain

(16) $P_{+F}, h'_{+F}, h \vdash L$ <u>except</u> $\mathcal{F}_S(F_{+F}, S)$

Therefore, combining (15) and (16) we obtain

6. is established.

1.2. Case (8) $e = e_1^{+F}.f$ (+F)

Therefore

(9) $e = e_1.\text{get-f}(\text{null}) \quad e_1 = \mathcal{C}(e_1^{+F})$

1.2.1 Case (10) $e_1^{+F} = L$

Then, with (10) and (9) we obtain

(11) $e_1 = L$

Define

(12) $Fs = \{f' \mid \exists \kappa_j \text{ with } \kappa_j = L.f'\}$

then, by (3), (12)

(13) $P_{+F}, h_{+F}, h \vdash L$ upto Fs

With (13) and CONG_OBJ, we obtain

(14) $h_{+F}(l) = [\![c \,\|\, qs \,\|\, fs]\!]$

(15) $h(l) = [\![c \,\|\, qs :: qs']\!]$

(16) $\text{dom}(qs') = \text{dom}(fs)$

(17) $\forall f' \in \text{dom}(qs') \cap Fs : qs'(f') = \emptyset$

(18) $\forall f' \in \text{dom}(qs') \setminus Fs : qs'(f') = \{fs(f')\}$

1.2.1.1 Case $f \in Fs$. Then $qs'(f) = \emptyset$, but this contradicts (3), because then the call $L.\text{get-f}(\text{null})$ would not be able to continue.

1.2.1.2 Case (19) $f \notin Fs$. Then, by (19), (18)

(20) $qs'(f) = \{ fs(f) \}$, let us call $v = fs(f)$

Therefore, with 3 and construction, we obtain

(21) $e' = L.f(v); v, \quad h' = h[L \mapsto C[qs : qs'[f \mapsto \phi]]]$

Using (21), (20) and CONG-GET we obtain

(22) $P_{tF}, h_{tF} \vdash e_{tF} \sim e'$ at $L.f$

Define

(23) $h'_{tF} = h_{tF}, \quad e'_{tF} = e_{tF}$

Using (20), (14)-(18) we obtain

(24) $P_{tF}, h_F, h \vdash L$ except $FSU\{f\}$

Define

(25) $K' = L.f$

Then, because $h_{tF}(L) \neq Upd$ and using CONG-OBJ-2, and CONG

(26) $\forall j \; K_j = L'$ except $\ldots \Rightarrow L \neq L'$

Therefore

(27) $\vdash K_1 \ldots K_n, K'$

Using CONG, (27), (23), (6) we obtain

6 is established.

Furthermore, taking $e'_{tF} = e_{tF}$, $h'_{tF} = h_{tF}$ gives

4 is established

5 is established using (22) and (25).

1.2.2 Case (10) $e_1^{+F} = \text{null}$

Then, both the SCHOOL and SCHOOL+F will execute to a null-pointer exception, not easy

1.2.2. Case (10) $e_1^{+F} = \text{widVal}$.

Contradiction, otherwise execution would be stuck.

1.2.3. Case (10) $e_1^{+F} = \text{nullPtrExc}$ Then both executions propagate the exception

1.2.4 Case (10) e_1^{+F} is a non-ground expression.

Therefore

(11) e_1 is a non-ground expression,

Therefore

(12) $e_1, h \rightsquigarrow e_1', h'$ and $e' = e_1'.f$

CONTradiction with the fact that derivation of (3) has depth 1. We will consider this in the inductive step.

1.3 Case (8) $e_{+F} = e_1^{+F}.f = e_2^{+F}$

Therefore,

(9) $e = e_1.\text{set-f}(e_2)$ and $e_1 = e(e_1^{+F})$ $e_2 = e(e_2^{+})$

1.3.1. Case (10) $e_1^{+F} = L$, $e_2^{+F} = V$

Then

(11) $e_1 = L$, $e_2 = V$

Define :

(12) $F_S = \{ f' \mid v_j = L.f' \text{ for some } j \in 1..n \}$

Then, by (3), (12) we obtain using CONG

(13) $P_{+F}, h_{+F}, h +_L$ upto F_S

With (13) and CONG-OBJ we obtain

(14) $h_{+F}(L) = [\![c \| qs \| fs]\!]$

(15) $h(L) = [\![c \| qs :: qs']\!]$

(16) $dom(fs) = dom(qs)$

(17) $\forall f' \in dom(qs') \cap F_S \quad qs'(f') = \emptyset$

(18) $\forall f' \in dom(qs') \setminus F_S \quad qs'(f') = \{ fs(f') \}$

By similar argument as in 1.2.1.1-12.1.2 we get

(19) $f \notin F_S$.

Therefore,

(20) $e' = L.f(v);$

(21) $h' = h[L \mapsto [\![c \| qs :: qs'[f \mapsto \emptyset]]\!]]$

Using (20) and (21) we obtain with CONG-SET

(22) $P_{+F}, h_{+F} \vdash e_{+F} \sim e'$ at $L.f$

Define

(23) $e'_{+F} = e_{+F}, \; h'_{+F} = h_{+F}$

Using similar steps to case 1.2.1.2, we obtain 4,5,6 is established.

1.3.2. Case $e_1^{T'} \in \{$ voidVal, null, nullPtrExc $\}$ similar to 1.2.2 – 12.4 cases

1.3.3 Case e_1^{+F} is not ground — will be dealt in the Inductive Step.

1.4 Case $e^{+F} \in \{$ L, null, voidVal $\}$

not applicable, because e^{+F} would be ground

1.5 Case $e^{+F} \neq$ nullPtrExc trivial

1.6 Case $e^{+F} = e_1^{+F}.m(e_2^{+F})$

1.6.1 Case (7) $e_1^{+F} = L$, $e_2^{+F} = v$. Then, we get

(8) $e = L.m(v)$

Furthermore, there exist c, qs'' so that

(9) $h(l) = [\![c \,\|\, qs'']\!]$

1st Case (10) $M(\Phi(P_{+F}), c, m) =$ async $m(-)$. Then by construction, (11) $M(P_{+F}, c, m) =$ async $m(-)$. Because of (10) we obtain

(12) $e' =$ voidVal $h' = h$

Now, define

(13) $e'_{+F} =$ voidVal, $h'_{+F} = h'$

Thus, with (11)

4 is established.

We define $k' = 0$

Then 5 is established.

Furthermore (2) and (12) and (13) give that

6 is established

1.6.2 Case $e_1^{+F} \in \{$ null, nullPE, voidVal $\}$ trivial

1.6.3 Case e_1^{+F}, or e_2^{+F} not ground belong to the inductive case

1.7 Case (8) $e_1^{+F} = e_1^{+F}; e_2^{+F}$.

1.7.1 Case (9) $e_1^{+F} = r$. Then

(10) $e = r; e_2$ where $e_2 = \varphi(e_2^{+F})$

Therefore, with (10) and (3)

(11) $e' = e_2$, $h' = h$

Take

(12) $e'_{+F} = e_2^{+F}$, $h'_{+F} = h_{+F}$, $k' = 0$

Then, we obtain

4, 5, 6 is established

1.7.2 Case e_1^{+F} is an exception trivial

1.7.3 Case e_1^{+F} not ground. Belongs to inductive step.

2nd Case CONG-VOID-VAL. Then

(7) $e \underset{tF}{=} v$, $e = voidVal; v$ $\kappa = 0$

Then, we obtain that

(8) $e' = v$; $h' = h$

Therefore, take $e'_{tF} = v$ $h'_{tF} = h_{tF}$ and then

4 is established
5 is established
6 is established.

3rd Case CONG-GET. Therefore

(7) $e \underset{tF}{=} L.f$, $e = L.ready\text{-}f(v); v$

(8) $\kappa = L.f$, $h_{tF}(l)\downarrow_3(f) = v$

We construct $Fs \in Id^t$, $Fs' \in Id^t$

(9) $Fs = \{f' \mid \exists kj: kj = L.f'\}$, $Fs' = Fs \cup \{f\}$

Then, because of (2), CONG, we obtain

(10) P_{tF}, h_{tF}, h_t L upto Fs'

Because of (10) and CONG-OBJ-2, there exist c, qs, fs, fs'...

(11) $h_{tF}(l) = [\![c \| qs \| fs]\!]$
(12) $h(l) = [\![c \| qs :: qs']\!]$
(13) $dom(qs') = dom\ fs$
(14) $f' \in dom(qs') \cap Fs'$: $qs'(f') = \emptyset$
(15) $f' \in dom(qs') \setminus Fs'$: $qs'(f') = \{fs(f)\}$

Because of (14) and (9) we obtain

(16) $h(\iota)\downarrow_2(f) = \emptyset$

Therefore, from (16) and (3) we obtain

(17) $e' = v$; voidVal

(18) $h' = h[L \mapsto [\![C \,\|\, qs :: qs'[f \mapsto \emptyset]]\!]]$

Now, define h'_{+F}, e'_{+F} as follows

(19) $h'_{+F} = h_{+F}$, $e'_{+F} = v$

Combine (19) with (18), (11), (12), (13), (14), (15) and CONG-OBJ-2, we obtain

(20) $P_{+F}, h'_{+F}, h' \vdash L$ upto Fs

(Note, that in 10 we had the large set, Fs', and here, in 20, only the set Fs.)

Now, take

(21) $k' = 0$

Then, use (19), (18), (20), (2), and using CONG we obtain

6 is established.

Using (19), (7) we obtain

4 is established

Using (21) and (19) (17) and CONG-VoidVal

5 is established.

4th Case CONG-SET.

Then

(7) $e_{+F} = L.f = v$, $e = L.\text{ready-}f(v)$

(8) $K = L.f$

We construct $F_S \subseteq \text{Idf}$, $F_S' \subseteq \text{Idf}$

(9) $F_S = \{f' \mid \exists K_j \; K_j = L.f'\}$, $F_S' = F_S \cup \{f\}$

Then, because of (2) and CONG, we obtain

(10) $P_{+F}, h_{+F}, h \vdash L$ upto F_S'

Because of (10), applying CONG-OBJ-2, there exist c, qs, qs', fs so that

(11) $h_{+F}(\iota) = [\![c \parallel qs \parallel fs]\!]$

(12) $h(\iota) = [\![c \parallel qs :: qs']\!]$

(13) $\text{dom}(qs') = \text{dom}(fs)$

(14) $f' \in \text{dom}(qs') \cap F_S' \Rightarrow qs'(f') = \emptyset$

(15) $f' \in \text{dom}(qs') \setminus F_S' \Rightarrow qs'(f') = \{fs(f)\}$

Because of (14) and (10) we obtain

(16) $h(\iota)\downarrow_2(f) = \emptyset$

Therefore, from (16) and (3) we obtain

(17) $e' = \text{voidVal}$;

5th Case CONG-NEW-1

Therefore

(7) $e^F = \text{new } c, \quad e = l.\text{init}(\text{null})$

(8) $K = l$ except $\mathcal{F}_s(P_{+F}, c)$

using (8) and (2) and CONG, CONG-OBJ-1, we obtain

(9) $h_{+F}(l) = \text{Udt}$

(10) $h(l) = [\![c \,\|\, qs :: qs']\!]$

(10a) $\text{dom}(qs) = \mathcal{U}^a(P_{+F}, c)$

(11) $\text{dom}(qs') = \mathcal{F}_s(P_{+F}, c)$

(12) $m \in \text{dom}(qs) \Rightarrow \quad qs(m) = \emptyset$

(13) $f \in \text{dom}(qs') \Rightarrow \quad qs'(f) = \emptyset$

By def of $\mathcal{C}(P_{+F})$ we obtain that

(14) $e' = L.f_1(\text{null}); \ldots L.f_n(\text{null}); L$

(15) $h' = h$

where

(16) $\mathcal{F}_s(P_{+F}, c) = \{f_1 \ldots f_n\}$

Define

(17) $e'_{+F} = \text{new } c \qquad h'_{+F} = h_{+F}, \; K' = K$

Thus,

4 is established from (17) and (7)

5 is established from (17), (9)-(13) CONG-NEW-1

6 is established from (2), (15), (17).

6th Case CONG-NEW-2

Therefore

(7) $e^F = \text{new } c,\quad e = L.f_1(\text{null}); \ldots L.f_n(\text{null}); L$

(8) $\kappa = L \text{ except } \{f_1 \ldots f_n\}$

Using (8) and (2) and CONG, and CONG-OBJ-1;

(9) $h_{tF}(l) = \text{Undf}$

(10) $h(L) = [\![c \| qs :: qs']\!]$

(11) $\text{dom}(qs) = M^a(P_{tF}, c)$

(12) $\text{dom}(qs') = \overline{F_L}(P_{tF}, c) \supseteq \{f_1 \ldots f_n\}$

(13) $m \in \text{dom}(qs) \Rightarrow qs(m) = \emptyset$

(14) $f \in \text{dom}(qs') \setminus \{f_1 \ldots f_n\} \Rightarrow qs(f) = \{\text{null}\}$

(15) $f \in \text{dom}(qs') \cap \{f_1 \ldots f_n\} \Rightarrow qs(f) = \emptyset$

By (7) and (2) and (11) and const of e...

We obtain

(20) $e' = L.f_2(\text{null}); \ldots L.f_n(\text{null}); L$

(21) $h' = h[L \mapsto [\![c \| qs :: qs'[f_1 \mapsto \{\text{null}\}]]\!]]$

Define

(22) $e'_{tF} = e_{tF},\quad h'_{tF} = h_{tF},\quad K' = L \text{ except } \{L_2, \ldots L_n\}$

Then

4 is established from 22

5 is established from (22), (21), 9-15 and CONG-NEW-1

6 is established from (9), (2), (22), (21) and CONG OBJ-2

7th Case CONG-CNTXT belong to inductive step.

Inductive Step

The depth of (1) is n+1. Therefore, the last rule applied in (1) is either CNTX or EX-PROP.

1st Case last rule is CNTX. Therefore, there exist a context E_0, and expressions e_0, e_1

(7) $e = E_0[e_0]$, $e_0, h \rightsquigarrow e_1, h'$, $e' = E_0[e_1]$

Furthermore, (7) and (1) imply that there exist SCHOL^{tF} expressions e_0^{tF}

(8) $e^{tF} = E_0[e_0^{tF}]$ and

(9) $P_{tF}, h \vdash e_0^{tF} \sim e_0$ at κ.

Applying the I.H. on (7), (9) and (2) we obtain, that there exist κ', h'_{tF}, e_1^{tF} so that

(10) $h'_{tF} = h_{tF}$, $e_1^{tF} = e_0^{tF}$ or $e_0^{tF}, h_{tF} \rightsquigarrow e_1^{tF}, h'_{tF}$

(11) $P_{tF}, h'_{tF} \vdash e_1^{tF} \sim e'_1$ at κ'

(12) $P_{tF} \vdash h'_{tF} \sim h'$ $\kappa_1, \kappa_2, \kappa'$

Define, (13) $e'^{tF} = E_0[e_1^{tF}]$. Then

(4) is established through (13), (10) and CNTX

(5) is established th (11) and (13) and CONG-CNTX

(6) is established thrg 12

2nd Case last rule was Ex-PROP. Then, both executions will propagate the exception. The exact argument along the lines of 1st case, however, here $\kappa' = 0$.