Lemma 6 If

(1) $P_{+F}, h_{+F} \vdash e_i^{+F} \sim e_i$ at κ_i

(2) $P_{+F} \vdash h_{+F} \sim h$ at $\kappa_1 .. \kappa_n$

(3) $e_1 .. e_n, h \rightsquigarrow e'_1 .. e'_n, h'$

then, there exist $e^{+F}_{n+1}, .. e^{+F}_{n+m}$ and h^{+F} and $\kappa'_1 .. \kappa'_m$, so that

(4) $e_1^{+F}, .. e_n^{+F}, h_{+F} \rightsquigarrow e^{+F}_{n+1}, .. e^{+F}_{n+m}, h'_{+F}$ or $h'_{+F} = h_{+F}$, $n=m$, $\forall i \in 1..n \;\; e_i^{+F} = e^{+F}_{n+i}$, $\kappa'_i = \kappa_i$

(5) $P_{+F}, h_{+F} \vdash e^{+F}_{n+1} \sim e'_i$ at κ'_i $\quad \forall i \in 1..m$

(6) $P_{+F} \vdash h'_{+F} \sim h'$ at $\kappa'_1, .. \kappa'_m$.

Proof By case analysis over $e_1 .. e_n, h \rightsquigarrow e'_1 .. e'_m, h$.

1st Case last rule applied was STRUNG. Then,

(7) $h(\iota) = [\![c \| qs]\!]$

(8) $AChs(\mathcal{P}(P_{+F}), c) \ni (\{m_1, .. m_k\}, e'')$

(9) $\forall i \in 1..n \quad qs(m_i) = q_i \cup \{v\}$

(10) $m = n+1$, $\forall i \in 1..n \quad e'_i = e_i$,

(11) $e'_{n+1} = e''[v_1/m_1\_x, v_2/m_2\_x, .. v_n/m_k\_x, this \mapsto \iota]$

(12) $h' = h[\iota \mapsto [\![c \| m_1 \mapsto q_1, .. m_n \mapsto q_n]\!]]$

From (8) and construction, we obtain, $\exists e'''_{+F}$

(13) $AChs(P_{+F}, c) \ni (\{m_1 .. m_k\}, e'''_{+F})$

(14) $e'' = \mathcal{P}(e'''_{+F})$

Using (2), and (7) we obtain

(15) $h_{+F}(\iota) = [\![c \parallel qs' \parallel fs]\!]$

(16) $dom(qs') \cup dom(fs) = dom(qs)$

Because of (8) $m_1 \ldots m_n$ are asynchronous methods, and therefore

(17) $m_1 .. m_n \in dom(qs')$.

By (16), (17), we obtain

(18) $qs'(m_i) = qs(m_i)$ $\forall i \in 1..k$

Defn (18a) $e^{+F}_{n+i} = e_i$ $i \in 1..n$, $m = n+1$

(19) $h'_{+F} = h_{+F}[\iota \mapsto [\![c \parallel qs'[m_1 \mapsto q_1, .. m_n \mapsto q_n] \parallel fs]\!]$

(20) $e^{+F}_{n+m} = e'''_{+F}[v_1/m_1, \ldots v_n/m_n]$

Then, by 19, 20, 15, 13 we obtain

(21) $e^{+F}_1, .. e^{+F}_n, h_{+F} \rightsquigarrow e^{+F}_{n+1}, .. e^{+F}_{n+n}, e^{+F}_{n+m}, h'_{+F}$

Furthermore, (14) and (11) and (20) together with CONG-BASIC give

(22) $P_{+F}, h'_{+F} \vdash e'_m \sim e^{+F}_{n+m}$ at 0

Taking $\kappa'_i = \kappa_i$ $i \in 1..n$, $\kappa'_m = 0$, we obtain

5, 6 are established

Also 4 is established from

2nd Case Last rule applied was PERM. This is basically a permutation. Easy.

3rd Case Last rule applied was RUN. Therefore

(7) $e_i' = e_i$ $i = 1..n-1$ and $n = m$

(8) $e_n, h \rightsquigarrow e_n', h'$

Applying lemma 5 on (8), (1) and (2); we get that there exist h'_{+F}, e'_{+F}, k' so that

(9) $e_{+F}, h_{+F} \rightsquigarrow e'_{+F}, h'_{+F}$ or $e'_{+F} = e_{+F}$, $h'_{+F} = h_{+F}$.

(10) $P_{+F}, h'_{+F} \vdash e'_{+F} \sim e'$ at k'

(11) $P_{+F} \vdash h'_{+F} \sim h'$ at $k_1, .. k_{n-1}, k'$

Define

(12) $e^{+F}_{n+i} = e^{+F}_i$ $i \in 1..n-1$ $k_n' = k'$

Then, we get

- 4 is established from 12, 7, 9 and (possibly) RUN
- 6 is established from 11 and 12

We need to show that 5 is also established.

Take any $i \in 1..n$. To show $P_{+F}, h_{+F} \vdash e^{+F}_{n+i} \sim e_i'$ at k'

1st Case $i = n$. Then follows from 10 and 12.

2nd Case $i < n$. Then, use CONG + get

(13) $\vdash k_1 .. k_{n-1}, k'$. Then (13) and (2), (12) give that

$P_{+F}, h'_{+F} \vdash e$