Theorem 1 If

(1) $\vdash P$,

(2) $P \vdash h$

(3) $e_1, \ldots e_n, h \rightsquigarrow e'_1 \ldots e'_m, h'$

(4) $P, h \vdash e_i : t_i$

$\Rightarrow$ $\exists\, t'_1 \ldots t'_m$, with

(5) $P \vdash h'$

(6) $P, h' \vdash e'_i : t'_i$ $\quad i = 1..m$

(7) $\{t_1 \ldots t_n\} \cup \{void\} = \{t'_1 \ldots t'_m\} \cup \{void\}$

Proof Case analysis over the last rule applied in 3.

1st Case PERM, trivial.

2nd Case RUN. Then

(8) $e_i = e'_i$ $\quad i = 1..n-1$

(9) $e_n, h \rightsquigarrow e'_n, h'$

Apply lemma 3 on 9, and get

5 is established

(10) $P, h' \vdash e'_n : t_n$

Furthermore, apply lemma 2 on 4, and get

(11) $P, h' \vdash e_i : t_i$ $\quad i = 1..n-1$

Then, define

(12) $t'_i = t_i$ $\quad i = 1..n$

Thus 6, 7 established from 12, 11.

3rd Case STRUNG. Then

(8) $h(\iota) = [\![c \,\|\, qs]\!]$

(9) $(\{m_1 \ldots n_n\}, e') \in \mathcal{A}Chr(P, c)$

(11) $qs(m_i) = \{v_i\} \cup q_i$

(12) $m \ldots n+1$, $e_i = e'_i$ $\quad i = 1..n$

(13) $e'_{n+1} = e'[v_1/m_1 \ldots v_n/m_n\text{-}x, \iota/this]$

(14) $h' = h[\iota \mapsto [\![c \,\|\, qs[m_1 \mapsto q_1, \ldots m_n \mapsto q_n]]\!]]$

Because of (9) and (1) $\exists t_i$

(14) $P, m_1 \_ x \mapsto t_1, \ldots m_n \_ x \mapsto t_n, \text{this} \mapsto c \vdash e'' : \text{void}$

(15) $\mathcal{M}(P, c, m_i) = \text{async } m_i(t_i)$

Using 11, 8, 2 we obtain

(16) $P, h \vdash v_i : t_i$

Therefore, using 13, 14, 16, and lemma 1 we obtain

(17) $P, h \vdash e'_{n+1} : \text{void}$

∴ Therefore

6 is established through 17, 12 and 4.

7 is established through 17

Finally,

5 is established from 2 and 14.