Theorem 2

We first prove the subject reduction property for one thread, ie,

(1) $P \vdash h$
(2) $\vdash P$
(3) $P, h \vdash e : t$
(4) $e, h \rightsquigarrow e', h'$

$\Rightarrow$ (5) $P \vdash h'$
(6) $P, h' \vdash e : t$

Proof by structural induction on (4), similar to the proof of lemma 3. We proceed by case analysis over the last step in (3). The cases CNTXT, EX-PROP, SEQ, EX, ASYNC and JOIN are known in the same way as in the proof of lemma 3. We now discuss the proof of the two new cases, ie. NEW, FLD-RD and FLD-WR.

1st Case NEW Then

(7) $e' = L$ where L new in h
(8) $e = \text{new } c$
(9) $h' = h[L \mapsto [\![c \,||\, m_1 \mapsto \emptyset \ldots m_n \mapsto \emptyset \,||\, f_1 \mapsto \text{null}, \ldots f_n \mapsto \text{null}]\!]]$
where
(10) $\{m_1 \ldots m_n\} = \mathcal{M}^a(P, c, m)$
(11) $\{f_1 \ldots f_n\} = \overline{\mathcal{F}}_s(P, c, m)$

Also, because of 3 we have that
(12) $P \vdash t \leq c$

Therefore
6 is established through 7, 9, RT-ADDR and RT-SUB.

Furthermore,

6 is established through 1, 11, 9, RT-NULL and FLD-WF-HEAP, and 2 (because that guarantees that the type of a field is a class type)

2nd Case FLD-RD Therefore

(7) $e = L.f$

(8) $h(l) = [\![c \| qs \| fs]\!]$

(9) $e' = fs(f)$

(10) $h' = h$

Therefore

5 is established from 1 and 5.

Furthermore, because of 3, we have that there exist C', C'' with

(11) $P \vdash C \leq C'$

(12) $\mathcal{F}(P, C', f) = C''$

(13) $P \vdash C'' \leq t$

Because (2) 11, 12 we obtain

(14) $\mathcal{F}(P, C, f) = C''$

Because of 1, 8, 9 we obtain

(15) $P, h \vdash e' : C''$

Therefore,

6 is established using 15, 13 and RT-SUB.

3rd Case FLD-WR Therefore,

(7) $e = L.f = v$

(8) $h(l) = [\![c \| qs \| fs]\!]$

(9) $e' = v$

(10) $h' = h[L \mapsto [\![c \| qs \| fs[f \mapsto v]]\!]]$

Because of (3) and (7) we obtain that there exists class c' and type t' with:

(11) $P \vdash c \leq c'$

(12) $P, h \vdash v : t'$

(13) $P \vdash t' \leq \mathcal{F}(P, c', f)$

(14) $P \vdash \mathcal{F}(P, c', f) \leq t$

Because of 12, 11, 13

(15) $\mathcal{F}(P, c, f) = \mathcal{F}(P, c', f)$

and therefore, with 15 and 13 and 12 obtain

(16) $P, h \vdash v : \mathcal{F}(P, c, f)$

Therefore,

5 is established through 1, 8, 10 and 16.

Furthermore,

6 is established through 12, 13, 14 and 15.

The proof for the multithreaded execution is identical to that of theorem 1.