Theorem 4 If

(1) $\Phi(P), \Gamma \vdash \Phi(e) : t$

then

(2) $P, \Gamma \vdash e : t$

Proof by structural induction on 1. Apply case analysis over the last rule applied in 1.

1st Case: NULL. Then $e = null = \Phi(e)$. Therefore, by application of NULL we obtain 2.

2nd Case VOID, ThisX, NEW, similar to 1st Case

3rd Case INV. Then, we obtain

(3) $\Phi(e) = e_1'.m(e_2')$

(4) $\Phi(P), \Gamma \vdash e_1' : c$

(5) $\mathcal{M}(\Phi(P), c, m) = t.\ m(t')$

(6) $\Phi(P), \Gamma \vdash e_2' : t'$

Because of 3 and definition of Φ there are four possibilities: either e was a method call, or a field read, or a field write, or an object creation

3.1 Case e is method call.

(7) $e = e_1.m(e_2)$

(8) $\Phi(e_1) = e_1'$

(9) $\Phi(e_2) = e_2'$

Because e is a SCHOOL+F expression, and because of our restriction on SCHOOL+F not to contain methods called get_, set_, init, ready_ we have

(10) $m \notin \{ get\_\ldots, set\_\ldots, init\_\ldots \}$

Therefore with (10) and (5) and definition of $\mathcal{P}$ we also obtain

(11) $\mathcal{M}(P, c, m) = t\ m(t')$

By application on I.H on (8), (4) we obtain

(12) $\mathcal{P}(P), \Gamma \vdash e_1 : c$

Similarly, by I.H on 6, 9 we obtain

(13) $\mathcal{P}(P), \Gamma \vdash e_2 : t'$

and so,

2 is established through INV, and 11, 12, 13

?

3.2 Case e is a field read, io

(7) $e = e_1.f$

(8) $\mathcal{P}(e_1) = e_1'$

(9) $m = get\_f$, $e_2' = null$

Apply I.H. on 7, 4 and obtain

(10) $P, \Gamma \vdash e_1 : c$

Because of (1) we obtain

(11.) $t\ get\_f(t') = M(\mathcal{P}(P), c, get\_f)$

because of def of $\mathcal{P}$ we also obtain

(12) $t = t'$

(13) $\mathcal{F}(P, c, f) = t$

Therefore

2 is established using 13, 10 and FLD-RD.

3.3 Case e is a field write; similar to 3.2.

3.4 Case e is an object creation. Similar to 3.2.