Cyclic abduction of inductive safety & termination preconditions

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Imperial College London, 24 Nov 2013

Joint work with Nikos Gorogiannis (Middlesex)

Part I

Introduction and motivations

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- Refined version: is my program safe and/or terminating, given that it satisfies some precondition?
- Even more refined version: can we find a reasonable precondition under which my program is safe and/or terminating?
- In this talk, we focus on this last question, using inductive definitions in separation logic to describe preconditions.

A simple example

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Most general solution is an acyclic linked list, given by

$$\begin{aligned} x = \mathsf{nil} \quad &\Rightarrow \quad \mathsf{list}(x) \\ x \neq \mathsf{nil} * x \mapsto y * \mathsf{list}(y) \quad &\Rightarrow \quad \mathsf{list}(x) \end{aligned}$$

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- …which means they must fail, or ask for advice, when encountering a "foreign" data structure.
- It would be nice if we could automatically infer the definitions of these data structures.

Overview of our approach

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- Implementation, CABER, builds on cyclic theorem prover CYCLIST:

  J. Brotherston, N. Gorogiannis, and R.L. Petersen.
  A generic cyclic theorem prover.
  In *APLAS* 2012.

Part II

Cyclic safety and termination proofs

Syntax of programs

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- Commands are those of a standard `while` language with pointers ($E.f$ refers to field f of expression E).
- A program is given by `fields` $n_1, \ldots, n_k$; C where each n_i is a field name and C a command sequence.

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- (C, s, h) is called safe if there is no computation sequence $(C, s, h) \rightsquigarrow^* \textit{fault}$. And $(C, s, h) \downarrow$ means there is no infinite computation sequence $(C, s, h) \rightsquigarrow \ldots$

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Proposition (Safety / termination monotonicity)

If (C, s, h) is safe and $h \circ h'$ defined then $(C, s, h \circ h')$ is safe.
If $(C, s, h) \downarrow$ and $h \circ h'$ defined then $(C, s, h \circ h') \downarrow$.

Preconditions

- Formulas F are given by

$$F \quad ::= \quad E = E \mid E \neq E \mid \mathsf{emp} \mid E \mapsto \mathbf{E} \mid P\mathbf{E} \mid F * F$$

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- Semantics given by standard forcing relation $s, h \models_{\Phi} F$

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- E.g., if C is $x := E.f; C'$, we have the symbolic execution rule:

$$\frac{x = \mathbf{E}_{\overline{f}}[x'/x] * (F * E \mapsto \mathbf{E})[x'/x] \vdash C'}{F * E \mapsto \mathbf{E} \vdash C} \quad |\mathbf{E}| \geq \overline{f}$$

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(Here, $\overline{f} \in \mathbb{N}$ and $\mathbf{E}_{\overline{f}}$ is the $\overline{f}$th element of $\mathbf{E}$. The variable x' is a fresh variable used to record the "old value" of x.)

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$$\begin{array}{rcl} x = \mathsf{nil} & \Rightarrow & \mathsf{bt}(x) \\ x \neq \mathsf{nil} * x \mapsto (y, z) * \mathsf{bt}(y) * \mathsf{bt}(z) & \Rightarrow & \mathsf{bt}(x) \end{array}$$

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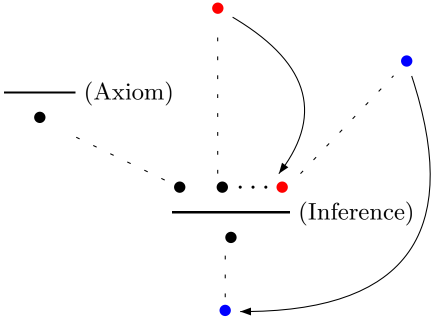
This gives the unfolding rule:

$$\frac{F * u = \mathsf{nil} \vdash C \qquad F * u \neq \mathsf{nil} * u \mapsto (y, z) * \mathsf{bt}(y) * \mathsf{bt}(z) \vdash C}{F * \mathsf{bt}(u) \vdash C}$$

Cyclic proofs

- A cyclic pre-proof is a derivation tree with back-links:



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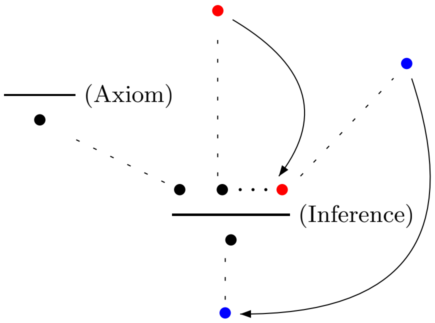

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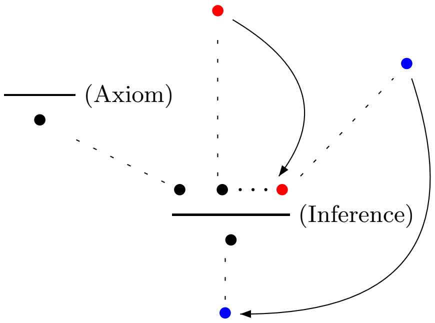

- Safety proof condition: there are infinitely many symbolic executions on every infinite path.
- Termination condition: some inductive predicate is unfolded infinitely often on every infinite path.

Soundness

Theorem

Fix rule set Φ, and program C, and suppose there is a cyclic proof $\mathcal{P}$ of $F \vdash C$. Let stack s and heap h satisfy $s, h \models_\Phi F$.

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- *If $\mathcal{P}$ satisfies the safety condition, (C, s, h) is safe;*
- *If $\mathcal{P}$ satisfies the termination condition, $(C, s, h) \downarrow$.*

Proof.

Inductive argument over proofs. $\square$

Part III

Cyclic abduction

Problem statement

- Problem: Given program C with input variables $\mathbf{x}$, find inductive rules Φ such that

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where P is a fresh predicate symbol, and "valid" may have either a safety or a termination interpretation.

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- Our approach: search for a cyclic safety/termination proof of $F \vdash C$, inventing inductive rules as necessary.

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Principle I (Proof search priorities)

Priority 1: *apply axiom rule*

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- *We may only abduce rules for undefined predicates.*
- *When we abduce rules for a predicate P in the current subgoal, we immediately unfold that predicate in the subgoal.*

We also need usual inductive theorem proving tricks like generalisation.

Worked example: binary tree search

$0:$ `while` $(x \neq \mathsf{nil})\{$
$1:$ `if`$(\star)$
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$3:$ `else`
$4: x := x.r \quad \}$
$5: \epsilon$

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5 : ϵ

$$\begin{aligned} x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x) \\ x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x) \\ x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x) \end{aligned}$$

$$\dfrac{\dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{l} x' \neq \mathsf{nil} * \\ x' \mapsto (x,z) * P_3(x',x,z) \end{array} \vdash 0}{\begin{array}{l} x \neq \mathsf{nil} * \\ x \mapsto (y,z) * P_3(x,y,z) \end{array} \vdash 2}\, x := x.l}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2) \qquad x \neq \mathsf{nil} * P_2(x) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)$$

Worked example: binary tree search

$$
\begin{array}{l}
0 : \texttt{while}\ (x \neq \mathsf{nil})\{ \\
1 : \texttt{if}(\star) \\
2 : x := x.l \\
3 : \texttt{else} \\
4 : x := x.r \quad \} \\
5 : \epsilon
\end{array}
$$

$$
\begin{array}{rl}
x = \mathsf{nil} * P_1(x) & \Rightarrow P_0(x) \\
x \neq \mathsf{nil} * P_2(x) & \Rightarrow P_0(x) \\
x \mapsto (y,z) * P_3(x,y,z) & \Rightarrow P_2(x) \\
P_0(y) * P_4(x,y,z) & \Rightarrow P_3(x,y,z)
\end{array}
$$

$$
\dfrac{
  \dfrac{
    \dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon
  }{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \begin{array}{l} x' \neq \mathsf{nil} * \\ x' \mapsto (x,z) * P_3(x',x,z) \end{array} \vdash 0
        }{
          \begin{array}{l} x \neq \mathsf{nil} * \\ x \mapsto (y,z) * P_3(x,y,z) \end{array} \vdash 2
        }\, x := x.l
      }{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
      \qquad
      x \neq \mathsf{nil} * P_2(x) \vdash 4
    }{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}
  }{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

0 : `while` $(x \neq \mathsf{nil})\{$
1 : `if`$(\star)$
2 : $x := x.l$
3 : `else`
4 : $x := x.r \quad \}$
5 : ϵ

$$\begin{aligned} x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x) \\ x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x) \\ x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x) \\ P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z) \end{aligned}$$

$$\dfrac{\dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\;\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\;\texttt{while} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\;\mathcal{A}(P_3)}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\;x := x.l}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\;\mathcal{A}(P_2) \qquad x \neq \mathsf{nil} * P_2(x) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\;\texttt{if}}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\;\texttt{while}}{P_0(x) \vdash 0}\;\mathcal{A}(P_0)$$

Worked example: binary tree search

$0:$ `while` $(x \neq \mathsf{nil})\{$
$1:$ `if`$(\star)$
$2: x := x.l$
$3:$ `else`
$4: x := x.r \quad \}$
$5: \epsilon$

$$
\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x) \\
x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x) \\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x) \\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}
$$

$$
\dfrac{
\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon
\over
x = \mathsf{nil} * P_1(x) \vdash 0
}{}\;\texttt{while}
\qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{P_0(x) \vdash 0}{\begin{array}{l} x' \neq \mathsf{nil} * \\ x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \end{array} \vdash 0}\,(\text{Frame})
}{\begin{array}{l} x' \neq \mathsf{nil} * \\ x' \mapsto (x,z) * P_3(x',x,z) \end{array} \vdash 0}\,\mathcal{A}(P_3)
}{\begin{array}{l} x \neq \mathsf{nil} * \\ x \mapsto (y,z) * P_3(x,y,z) \end{array} \vdash 2}\,x := x.l
}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
\qquad x \neq \mathsf{nil} * P_2(x) \vdash 4
}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}
}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
$$

$$
\dfrac{x = \mathsf{nil} * P_1(x) \vdash 0 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 0}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

$0:$ `while` $(x \neq \mathsf{nil})\{$
$1:$ `if`$(\star)$
$2: x := x.l$
$3:$ `else`
$4: x := x.r \quad \}$
$5: \epsilon$

$$
\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x)\\
x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}
$$

$$
\dfrac{
\dfrac{
\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon
}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while}
\qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\,\text{(Frame)}
}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\,\mathcal{A}(P_3)
}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\,x := x.l
}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
\qquad
x \neq \mathsf{nil} * P_2(x) \vdash 4
}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}
}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

0 : `while` $(x \neq \mathsf{nil})\{$
1 : `if`$(\star)$
2 : $x := x.l$
3 : `else`
4 : $x := x.r \quad \}$
5 : ϵ

$$
\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x) \\
x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x) \\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x) \\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}
$$

$$
\dfrac{
  \dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\;\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\;\texttt{while}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \dfrac{
            \dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\;(\text{Frame})
          }{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\;\mathcal{A}(P_3)
        }{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\;x := x.l
      }{x \neq \mathsf{nil} * P_2(x) \vdash 2}\;\mathcal{A}(P_2)
      \qquad
      \dfrac{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 4}\;(P_2)
    }{x \neq \mathsf{nil} * P_2(x) \vdash 1}\;\texttt{if}
  }{x \neq \mathsf{nil} * P_2(x) \vdash 0}\;\texttt{while}
}{P_0(x) \vdash 0}\;\mathcal{A}(P_0)
$$

Worked example: binary tree search

$0:$ `while` $(x \neq \mathsf{nil})\{$
$1:$ `if`$(\star)$
$2: x := x.l$
$3:$ `else`
$4: x := x.r \quad \}$
$5: \epsilon$

$$
\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x)\\
x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}
$$

$$
\dfrac{
  \dfrac{
    \dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon
  }{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \dfrac{
            \dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\,(\text{Frame})
          }{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\,\mathcal{A}(P_3)
        }{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\,x := x.l
      }{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
      \qquad
      \dfrac{
        \dfrac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}\,x := x.r
      }{x \neq \mathsf{nil} * P_2(x) \vdash 4}\,(P_2)
    }{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}
  }{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

$0:\texttt{while}\ (x \neq \mathsf{nil})\{$
$1:\texttt{if}(\star)$
$2: x := x.l$
$3:\texttt{else}$
$4: x := x.r \quad \}$
$5: \epsilon$

$$
\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x)\\
x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}
$$

$$
\dfrac{
  \dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \dfrac{
            \dfrac{P_0(x) \vdash 0}{\begin{array}{l} x' \neq \mathsf{nil} * \\ x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \end{array} \vdash 0}\,(\text{Frame})
          }{\begin{array}{l} x' \neq \mathsf{nil} * \\ x' \mapsto (x,z) * P_3(x',x,z) \end{array} \vdash 0}\,\mathcal{A}(P_3)
        }{\begin{array}{l} x \neq \mathsf{nil} * \\ x \mapsto (y,z) * P_3(x,y,z) \end{array} \vdash 2}\,x := x.l
      }{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
      \quad
      \dfrac{
        \dfrac{\begin{array}{l} x' \neq \mathsf{nil} * \\ x' \mapsto (y,x) * P_3(x',y,x) \end{array} \vdash 0}{\begin{array}{l} x \neq \mathsf{nil} * \\ x \mapsto (y,z) * P_3(x,y,z) \end{array} \vdash 4}\,x := x.r
      }{x \neq \mathsf{nil} * P_2(x) \vdash 4}\,(P_2)
    }{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}
  }{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

$$
\begin{array}{l}
0 : \texttt{while}\ (x \neq \mathsf{nil})\{\\
1 : \texttt{if}(\star)\\
2 : x := x.l\\
3 : \texttt{else}\\
4 : x := x.r \quad \}\\
5 : \epsilon
\end{array}
\qquad
\begin{array}{rl}
x = \mathsf{nil} * P_1(x) & \Rightarrow P_0(x)\\
x \neq \mathsf{nil} * P_2(x) & \Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) & \Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) & \Rightarrow P_3(x,y,z)
\end{array}
$$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{ϵ}
\UnaryInfC{$x = \mathsf{nil} * P_1(x) \vdash 5$}
\RightLabel{\texttt{while}}
\UnaryInfC{$x = \mathsf{nil} * P_1(x) \vdash 0$}
\AxiomC{$P_0(x) \vdash 0$}
\RightLabel{(Frame)}
\UnaryInfC{$x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0$}
\RightLabel{$\mathcal{A}(P_3)$}
\UnaryInfC{$x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0$}
\RightLabel{$x := x.l$}
\UnaryInfC{$x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2$}
\RightLabel{$\mathcal{A}(P_2)$}
\UnaryInfC{$x \neq \mathsf{nil} * P_2(x) \vdash 2$}
\AxiomC{$x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_4(x',y,x) \vdash 0$}
\RightLabel{(P_3)}
\UnaryInfC{$x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0$}
\RightLabel{$x := x.r$}
\UnaryInfC{$x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4$}
\RightLabel{(P_2)}
\UnaryInfC{$x \neq \mathsf{nil} * P_2(x) \vdash 4$}
\RightLabel{\texttt{if}}
\BinaryInfC{$x \neq \mathsf{nil} * P_2(x) \vdash 1$}
\RightLabel{\texttt{while}}
\UnaryInfC{$x \neq \mathsf{nil} * P_2(x) \vdash 0$}
\RightLabel{$\mathcal{A}(P_0)$}
\BinaryInfC{$P_0(x) \vdash 0$}
\end{prooftree}
$$

Worked example: binary tree search

$0:$ `while` $(x \neq \mathsf{nil})\{$
$1:$ `if`$(\star)$
$2: x := x.l$
$3:$ `else`
$4: x := x.r \quad \}$
$5: \epsilon$

$$
\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x)\\
x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)\\
P_0(z) * P_5(x,y,z) &\Rightarrow P_4(x,y,z)
\end{aligned}
$$

$$
\dfrac{
  \dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon
  \;\;\text{while}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \dfrac{
            \dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\,(\text{Frame})
          }{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\,\mathcal{A}(P_3)
        }{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\,x := x.l
      }{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
      \quad
      \dfrac{
        \dfrac{
          \dfrac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_4(x',y,x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0}\,(P_3)
        }{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}\,x := x.r
      }{x \neq \mathsf{nil} * P_2(x) \vdash 4}\,(P_2)
    }{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\text{if}
  }{}
}{
  x = \mathsf{nil} * P_1(x) \vdash 0 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 0
}\,\text{while}
$$

$$
\dfrac{x = \mathsf{nil} * P_1(x) \vdash 0 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 0}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

$0 : \texttt{while}\ (x \neq \mathsf{nil})\{$
$1 : \texttt{if}(\star)$
$2 : x := x.l$
$3 : \texttt{else}$
$4 : x := x.r \quad \}$
$5 : \epsilon$

$$
\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x)\\
x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)\\
P_0(z) * P_5(x,y,z) &\Rightarrow P_4(x,y,z)
\end{aligned}
$$

$$
\dfrac{
\dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while}
\qquad
\dfrac{\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\,(\mathrm{Frame})}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\,\mathcal{A}(P_3)}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\,x := x.l}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_0(x) * P_5(x',y,x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_4(x',y,x) \vdash 0}\,\mathcal{A}(P_4)}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0}\,(P_3)}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}\,x := x.r}{x \neq \mathsf{nil} * P_2(x) \vdash 4}\,(P_2)
}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

$0:$ `while` $(x \neq \mathsf{nil})\{$
$1:$ `if`$(\star)$
$2: x := x.l$
$3:$ `else`
$4: x := x.r \quad \}$
$5: \epsilon$

$$\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x)\\
x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)\\
P_0(z) * P_5(x,y,z) &\Rightarrow P_4(x,y,z)
\end{aligned}$$

$$\dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\ (\text{Frame})$$

$$\dfrac{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\ \mathcal{A}(P_3)$$

$$\dfrac{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\ x := x.l$$

$$\dfrac{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\ \mathcal{A}(P_2)$$

$$\dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_0(x) * P_5(x',y,x) \vdash 0}\ (\text{Frame})$$

$$\dfrac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_0(x) * P_5(x',y,x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_4(x',y,x) \vdash 0}\ \mathcal{A}(P_4)$$

$$\dfrac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_4(x',y,x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0}\ (P_3)$$

$$\dfrac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}\ x := x.r$$

$$\dfrac{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 4}\ (P_2)$$

$$\dfrac{x \neq \mathsf{nil} * P_2(x) \vdash 2 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\ \texttt{if}$$

$$\dfrac{x \neq \mathsf{nil} * P_2(x) \vdash 1}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\ \texttt{while}$$

$$\dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\ \epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\ \texttt{while}$$

$$\dfrac{x = \mathsf{nil} * P_1(x) \vdash 0 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 0}{P_0(x) \vdash 0}\ \mathcal{A}(P_0)$$

Worked example: binary tree search

0 : `while` $(x \neq \mathsf{nil})\{$
1 : `if`$(\star)$
2 : $x := x.l$
3 : `else`
4 : $x := x.r \quad \}$
5 : ϵ

$$
\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x)\\
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\qquad
\frac{x \neq \mathsf{nil} * P_2(x) \vdash 1}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\;\texttt{while}$$

$$\frac{x \neq \mathsf{nil} * P_2(x) \vdash 2 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\;\texttt{if}$$

$$\frac{\dfrac{\dfrac{\dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\;(\text{Frame})}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\;\mathcal{A}(P_3)}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\;x := x.l}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\;\mathcal{A}(P_2)$$

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_0(x) * P_5(x',y,x) \vdash 0}\;(\text{Frame})}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_4(x',y,x) \vdash 0}\;\mathcal{A}(P_4)}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0}\;(P_3)}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}\;x := x.r}{x \neq \mathsf{nil} * P_2(x) \vdash 4}\;(P_2)$$

Simplifying inductive rule sets

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Simplifying inductive rule sets

- instantiate undefined predicates to emp;

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(nil-terminated binary tree)

Simplifying inductive rule sets

- instantiate undefined predicates to emp;
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- remove unsatisfiable clauses (not shown)

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Part IV

Challenges and subtleties

Evaluating solution quality

- Backtracking search can yield different solutions.

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- We can decide whether a predicate is satisfiable

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- Comparing predicates via entailment ($\vdash$) is not practical.
- Currently we use a simple grading scheme for predicate quality.
- We can simplify predicates and replay the proof to improve quality, sometimes.

Experimental results

Program	LOC	Time	Depth	Quality	Term.
List traverse	3	20	3	A	✓
List insert	14	8	7	B	✓
List copy	12	0	8	B	✓
List append	10	12	5	B	✓
Delete last from list	16	12	9	B	✓
Filter list	21	48	11	C	✓
Dispose list	5	4	5	A	✓
Reverse list	7	8	7	A	✓
Cyclic list traverse	5	4	5	A	✓
Binary tree search	7	8	4	A	✓
Binary tree insert	18	4	7	B	✓
List of lists traverse	7	8	5	B	✓
Traverse even-length list	4	8	4	A	✓
Traverse odd-length list	4	4	4	A	✓
Ternary tree search	10	8	5	A	✓
Conditional diverge	3	4	3	B	×
Traverse list of trees	11	12	6	B	✓
Traverse tree of lists	17	68	7	A	✓
Traverse list twice	8	64	9	B	✓

Problem: initial variable assignment

- Consider a local variable assignment $y := x$ at line 0. In the proof we get

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- But there are lots of choices!

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- In principle, we could also use the control flow graph of the program to help us decide what to do.

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- Here it shows up in the need for lemmas:

$$\frac{\Pi : F * \mathsf{list}(x) \vdash i}{\Pi : F * x \mapsto y \vdash i} \quad x \mapsto y \vdash \mathsf{list}(x) \quad \text{(Cut)}$$

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- Our tool has a limited abstraction capability, mainly based on existentially quantifying variables modified by loops.
- Lemma speculation is a well known problem in inductive theorem proving. In our setting, where parts of the lemma may be undefined, it is harder still!
- CYCLIST gives us an entailment prover which could be used to prove conjectured lemmas.

Thanks for listening!

Get Caber / Cyclist online (source / virtual machine image):

google "cyclist theorem prover".

J. Brotherston and N, Gorogiannis.
Cyclic abduction of inductive safety and termination preconditions.
Submitted.