Forms of factoring in paramodulation-based calculi

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Abstract

It is a well-known fact that some form of factoring is necessary for completeness of paramodulation-based calculi of general first-order clauses. In this paper we give an overview of some existing formalizations of factoring, their related paramodulation-based calculi, and corresponding completeness results. We also propose a factored superposition rule that "combines" usual superposition and factoring in a way similar to the rule presented in Moser et al. (1995). The advantage of our factored superposition rule is that it can be used in methods that combine bottom-up and top-down reasoning (like equality elimination for tableau method, see Degtyarev and Voronkov (2001)) without compromising completeness in presence of (the basic version of) tautology deletion and subsumption rules.

1 Ordered factoring

We first consider the *o-factoring* rule, introduced in Hsiang and Rusinowitch (1986) as an inference rule of their ordered paramodulation calculus.

If $L_1, \ldots, L_k$ are literals of a clause C and are unifiable with mgu σ, and for every other atom $A \in C, L_1\sigma \not\preceq A\sigma$, then $D = C\sigma - \{L_2\sigma, \ldots, L_k\sigma\}$ is an o-factor of C.

The restrictions that the authors pose to applying the rule is that "it is only applied to clauses on which some inference is about to be applied". Ordered paramodulation was proved, using the transfinite semantic tree method, refutationally complete with no other restrictions to applications of the inference rules, and, in particular, without presence of deletion rules.

Pais and Peterson (1991) define the *maximal factoring inference rule* (essentially the same rule as o-factoring). They further constraining the paramodulation rule to the form known as *maximal paramodulation*, and show completeness of the corresponding calculus using the forcing technique. This technique was powerful enough to allow for deletion rules, like simplification, subsumption, and tautology deletion.

Bachmair and Ganzinger (1990) define *positive factoring* inference rule, which is a restriction of the maximal factoring (o-factoring) rule to positive literals. They show, by the model generation method, that the corresponding ordered paramodulation calculus is refutationally complete even in presence of deletion rules like subsumption, simplification, and tautology elimination. In Bachmair et al. (1992) they extend their result for ordered paramodulation to the basic setting, and also define basic superposition. However, they discover that basic superposition in presence of ordered factoring is not complete, and hence extend the calculus by another rule, so-called *merging paramodulation*.

The first result that shows completeness of basic superposition with the ordered factoring rule is Bachmair and Ganzinger (1998). However, their *strict basic superposition* calculus suffers when it comes to redundancy elimination. Namely, it is compatible only with so-called weak redundancy elimination. As a result, only weak tautologies can be removed, which is a criterion not strong enough to cover all tautologies.

2 Equality factoring

Nieuwenhuis and Rubio (1992) define *equality factoring* as "a generalization to the equality case of "normal" factoring'.

$$\frac{\Gamma, s \approx t, s' \approx t'}{\Gamma, t \not\approx t', s \approx t}$$

where $s = s'$, $s \approx t$ is the maximal literal in the premise, and $s \succ t, s' \succ t'$.

The link with "normal" factoring (factoring used in resolution, and ordered factoring) is clear: if s and s' are atoms, then both t and t' can be the symbol $true$ and the negative equation $true \approx true$ can be omitted from the conclusion.

Nieuwenhuis and Rubio show that basic superposition with equality factoring is complete with basic subsumption, basic simplification and elimination of tautologies in the basic setting. Because of compatibility with these redundancy criteria, equality factoring was later also used by Bachmair et al. in Bachmair et al. (1995), who favoured the rule over the combination of ordered factoring and merging paramodulation. It should be noted though, that the equality factoring rule is not very appealing: it combines ordered factoring with resolution into a certain instance of the transitivity axiom.

In order to improve efficiency of goal-directed methods (like model elimination), different authors (see Moser et al. (1995); Degtyarev and Voronkov (1996)) suggested combining them with the bottom-up saturation (by a paramodulation-based calculus) of the original formula (goal). A special approach for dealing with general clauses with respect to equality literals (Degtyarev and Voronkov (1996) address only Horn clauses w.r.t. equality literals, and therefore do not consider factoring) is the one taken by Moser et al. (1995). Because of incompatibility of "the most of refinements of paramodulation with the goal-directed paradigm", they propose the *basic factored paramodulation calculus*. The main feature of this calculus is that its paramodulation rule contains factoring, which is hence not defined as a separate inference rule. Although the calculus is complete with basic simplification, basic subsumption and tautology removal in the basic setting, it looses these nice features when combined with the model elimination method.

In order to overcame this drawback, we suggest another approach, where superposition and factoring are contained within the following *basic factored overlap* rule:

$$\frac{\Gamma_1, l_1 \approx r_1, \ldots, l_m \approx r_m \mid T_1 \quad \Gamma_2, s[l'] \approx t \mid T_2}{\Gamma_1, \Gamma_2, s[x] \approx t, x \not\approx r_1, \ldots, x \not\approx r_m \mid T_1 \wedge T_2 \wedge \delta}$$

where x is a fresh variable and δ stands for $s \succ t \wedge l_1 \succ r_1 \wedge \ldots \wedge l_m \succ r_m \wedge l_1 = l' \wedge \ldots \wedge l_m = l'$.

Opposite to the basic factored overlap rule of Moser et al. (1995), which postpones factoring until the end of the proof, our rule "converts" factoring into equality solution. Equivalently, our rule always transforms unifiable positive literals into unifiable negative literals, and thus truly avoids factoring. The advantage of having such a rule is that it can be combined with goal-directed procedures, like equality elimination and basic folding without losing compatibility with redundancy elimination. In Aleksić (2007), we show that basic folding (which incorporates our basic factored overlap rule) is complete with elimination of tautologies (in the basic setting) and basic subsumption.

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