SAN models of optical networks

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Abstract

In this paper we discuss a modelling of an optical telecommunication network with an analytical tool. We present a Markovian model of an edge node which is an interface between electronic and optical worlds responsible for the optical medium access control. The formal tool applied to the generator matrix construction is the Stochastic Automata Network formalism. Thanks to this compositional approach we are able to express the behaviour of the complex computer network in the analytical terms.

1 Markovian theory versus modelling practice

In three years Markovian processes will celebrate the 100th anniversary of their discovery. A.A. Markov nabbed them in a beautiful (and rather long) poem (in Russian) when analysing the alternation of consonants and vowels. It turned out that a lot of events taking place nature have something in common with a two-state consonant-vowel literary process: memoryless. Shortly after their revelation, Markov processes found applications in many disciplines such as thermodynamics, card shuffling, and telephony (Erlang). As we see, Markov chains[1] have been placed like a horse to cart to the modelling of information exchange systems since their youth.

As computer systems and networks become more complex, Markovian methods have to be adapted to cope with their technical complexity. Three major problems which have been solved are as follows: 1) Generation of elements of transition/stochastic matrix, 2) Storage of elements of this matrix, 3) Numerical solution of the linear system. A modeller stands up to all of them in order to master time and space constraints which their impose.

In this paper we work on a Markovian model of an optical network ring. Its architecture and medium access protocol are complex and the construction of the generator matrix is a serious task. In order to compute

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[1] process with discrete state space

elements of the matrix we apply the Stochastic Automata Network (SAN) formalism (introduced in [3], presented also e.g. in [1, 4, 5, 7]) which allows us to see a modelling Markovian chain as a set of cooperating unidimensional processes. To build the model containing technological details, we base our approach upon synchronisation transitions with functional rates.

In the next section we give a description of the network to be modelled. Afterwards we show in what way the optical ring was expressed by means of SAN. The final section contains remarks on the comparison of obtained results with those estimated by simulation. We will also include there some notes concerning analytical time modelling of the ring network.

2 Optical network architecture

The concept of all-optical networks [2] is being intensely researched on as the prospective technology for the future Internet. This technology is also investigated as a mean of the migration of packets from Local Area Networks (LAN) to Metropolitan Area Networks (MAN).

We want to model a double bus all-optical metro ring network architecture [6] based on variable-length optical packet-switching technology [10], Figures 1 and 2.

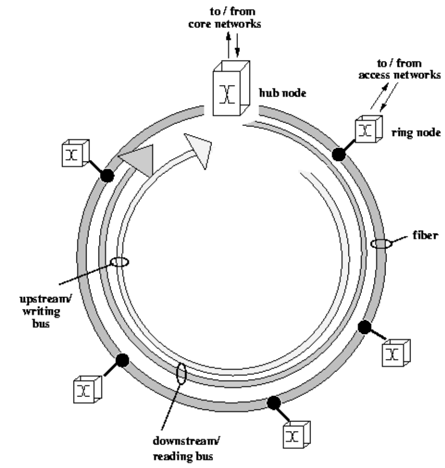

Figure 1: Double Bus Optical Ring Network

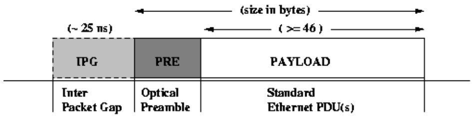

Figure 2: Optical packet format

3 Markovian model of an edge node as an SAN

We consider our model as a Markov chain in continuous time (CTMC) because the length of optical packets is not fixed and because IP packets entering the optical ring may vary in length.

3.1 General model description

We build our model for the i^{th} station on the n-station ring, $i = 1, 2, \ldots, n$. To cope with the problem of electronic IP packets of different sizes, we divide them into three classes. The first type of packets, with a mean size of $m_1 = 50$ Bytes, includes the smallest ones which primarily transport signalization and control data. We take the length of the shortest packet as the length unit and we call it u. The medium packets, with a mean size of $m_2 = 500$ Bytes $= 10u$, form the second class. The last type of packets, with a mean size of $m_3 = 1500$ Bytes $= 30u$, contains the Ethernet MTU (Maximum Transmission Unit). We model these three packet streams through exponential sources (Fig. 3) with generation rates (γ_1, γ_2, γ_3).

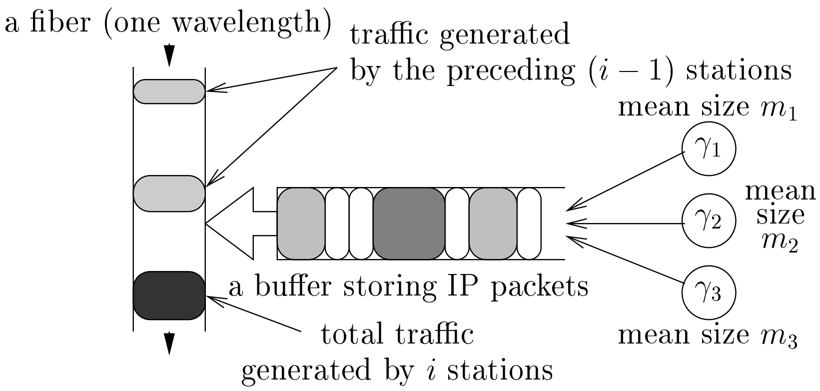

Figure 3: An edge node in the i^{th} station of a ring — IP packets, waiting in a FIFO buffer, try to enter a fiber, varied optical packet size

Packets of all classes try to access a wavelength which may be occupied by packets sent by the preceding $(i-1)$ stations which are still being transmitted (only the first station can always take over the optical medium). If the wavelength is busy, electronic packets are waiting for access in a buffer with FIFO scheduling. The i^{th} station is permanently observing the wavelength. When the optical medium becomes free, the station starts to measure a time of the current free period. Let us note a time length needed for an optical transmission of the data unit u as t_u. If a packet of size αu, $\alpha = 1, 10, 30$ is to be transmitted, the station must be sure that there will be enough space for it on the fiber (αt_u, $\alpha = 1, 10, 30$). If the station states that there is enough room for the packet first to be transmitted, it sends it and keeps observing the wavelength. If an available period on the fiber is longer than $30t_u$ (the window length; equal to MTU) and the station does not have packets to send, a time counter is reset and the station resumes measuring a next free period on the wavelength.

3.2 Representation of the model in the SAN formalism

A state of the modelling CTMC is composed of four elements: $(s^{(0)}, s^{(1)}, s^{(2)}, s^{(3)})$. The component $s^{(0)}$ shows if the observed wavelength is occupied or not. If the wavelength is free, it indicates the length of the period available for transmission. The following components $s^{(1)}, s^{(2)}, s^{(3)}$ represent the numbers of packets of type k, waiting in the buffer. Notice that we do not store the order with which the packet are placed in the buffer. We determine a class of a packet to be sent by a probability p_k, $k = 1, 2, 3$, that a packet of class k is at the head of the buffer: $p_k = \frac{s^{(k)}}{\sum_{i=1}^{3} s^{(i)}}$. We note that the state element $s^{(k)}$ is controlled by the automaton $\mathrm{SA}^{(k)}$.

The local transitions for $\mathrm{SA}^{(0)}$ (Fig. 4 for the stations i, $i = 2, 3, \ldots, n$, Fig. 5 for the first station on the ring which can always access the optical medium) occur when the wavelength changes its free/busy status. The wavelength becomes busy with a transition rate λ and it becomes free with a rate μ, both exponential. Measurement of a space available on the wavelength is realized in $\mathrm{SA}^{(0)}$ by local transitions with an exponential rate τ_1. A single transition with τ_1 corresponds to a time unit t_u, $\tau_1 = \frac{1}{t_u}$.

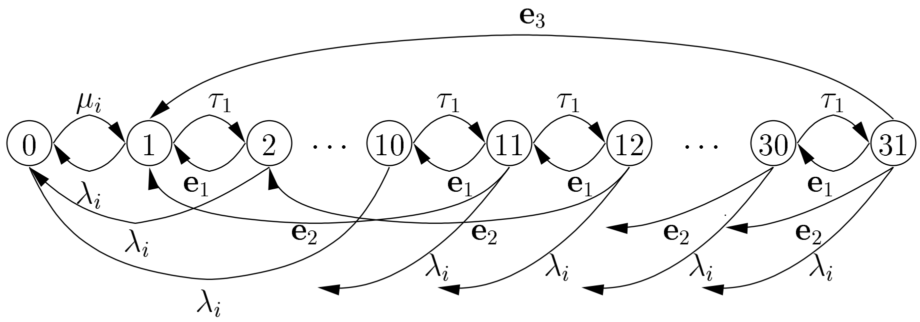

Figure 4: The automaton $\mathrm{SA}^{(0)}$ representing the fiber state seen by the i^{th} station, $i = 2, 3, \ldots, n$; 0 state indicates that the wavelength is busy, 1 indicates that it is free, an observation window length is set for 30 units, $\mathbf{e}_k = (e_k, \theta, p_k \mathcal{C}_k)$, $k = 1, 2, 3$, where e_k — a stochastic event indicating transmission of a packet of class k on the fiber

Local transitions in the automata $\mathrm{SA}^{(k)}$, $k = 1, 2, 3$, happen when a packet of class k is generated with a rate of source γ_k (Fig. 6).

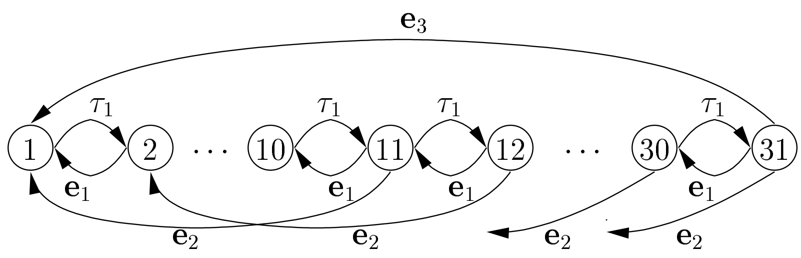

Figure 5: The automaton $\mathrm{SA}^{(0)}$ representing the fiber state seen by the first station $i = 1$; the wavelength is always available for transmissions, an observation window length is set for 30 units, $\mathbf{e}_k = (e_k, \theta, p_k\mathcal{C}_k)$, $k = 1, 2, 3$, where e_k — a stochastic event indicating transmission of a packet of class k on the fiber

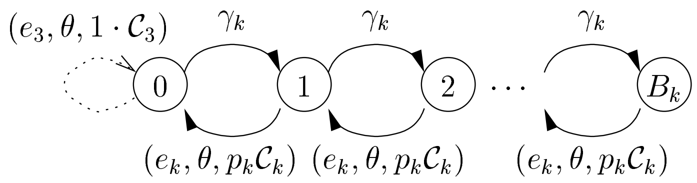

Figure 6: The automata $\mathrm{SA}^{(k)}$, $k = 1, 2, 3$, representing the number of packets of each type in the buffer; B_k — buffer capacity is expressed as the maximal number of type k packets which can be stored in it, e_k — a stochastic event indicating transmission of a packet of type k on the optical fiber (a dotted loop is valid for the $\mathrm{SA}^{(3)}$ only)

The automaton of time measuring, $\mathrm{SA}^{(0)}$, and the automata of the buffer, $\mathrm{SA}^{(k)}$, $k = 1, 2, 3$, must cooperate in case of transmission of an IP packet. These interactions are introduced into the model as synchronization events $\mathbf{e}_k$, $k = 1, 2, 3$: the $\mathbf{e}_k$ event is responsible for a transmission of a packet of class k and takes place in two automata, $\mathrm{SA}^{(0)}$ and $\mathrm{SA}^{(k)}$ at the same time. The complete description of $\mathbf{e}_k$ is given by a triple $(e_k, \theta, p_k\mathcal{C}_k)$, where θ is a transmission rate which corresponds to the delay needed by a station to detect if a wavelength is free, p_k is a probability that a packet of class k is first to be sent (this has been already discussed above), and $\mathcal{C}_k$ is a firing condition, i.e. a condition under which the given event may happen. The events $\mathbf{e}_1$ and $\mathbf{e}_2$ are always enabled, $\mathcal{C}_1 = \mathcal{C}_2 = 1$. The event $\mathbf{e}_3$ may be fired or for a transmission of a packet, $\mathcal{C}_2 = 1$, as $\mathbf{e}_1$ and $\mathbf{e}_2$ may, or to terminate an observation window if there nothing to be sent. Notice that in this very case we put $p_3 = 1$ (see Fig. 6 in which this option is marked with dotted loop).

Let us note B as a number of places in the buffer expressed in unit u and $\alpha_1 u$, $\alpha_2 u$, $\alpha_3 u$, where $\alpha_1 = 1$, $\alpha_2 = 10$, $\alpha_3 = 30$ are mean sizes of packets of each of three packet types (see Subsection 3.1). The maximal packet numbers of each class which may be stored in the buffer are equal to $B_1 = \frac{B}{\alpha_1} = B$, $B_i = \frac{B}{\alpha_i}$, $i = 2, 3$. A number of packets waiting in the buffer cannot exceed its total capacity, $\alpha_1 s^{(1)} + \alpha_2 s^{(2)} + \alpha_3 s^{(3)} \leq B$. This inequality allows us to compute a number of possible configurations of electronic packets in the buffer. Let us take firstly the component $s^{(1)}$. The number of configurations is $\mathcal{N}_1(B) = B_1 + 1 = B + 1$. For the two elements $s^{(1)}$ and $s^{(2)}$ the number of permissible arrangements is given by $\mathcal{N}_2(B) = \sum_{i=0}^{B_2} \mathcal{N}_1(B - i\alpha_2) = (B+1)(B_2+1) - \frac{1}{2}\alpha_2 B_2(B_2+1) = (\frac{B}{\alpha_2}+1)(\frac{B}{2}+1)$. Analogously, for three class components, $s^{(1)}$, $s^{(2)}$, $s^{(3)}$, we obtain $\mathcal{N}_3(B) = \sum_{i=0}^{B_3} \mathcal{N}_2(B - i\alpha_3)$ and this formula may be generalized for any number of components:

$$\mathcal{N}_m(B) = \sum_{i=0}^{B_m} \mathcal{N}_{m-1}(B - i\alpha_m).$$

We notice that any state of $\mathrm{SA}^{(0)}$ ($|S^{(0)}| = 32$) is possible for any configuration of states of $\mathrm{SA}^{(i)}$, $i = 1, 2, 3$. Therefore, we can compute the number of attainable states of the global Markov chain: $\mathcal{N}(B) = 32\mathcal{N}_3(B)$.

4 Conclusions

The Markov chain describing the behaviour of an edge node has been solved for different sets of parameters and its results have been compared with those obtained by a custom-made simulation tool. We have to make clear at this point that the Markovian approach did not allow us to study systems with a buffer size like in real life. In order to keep the Markov matrix dimension under a reasonable limit, we restricted the buffer size. Technical details considering the technological parameters and the results the reader can find in [9]. They show that we have modelled "the same system" with different approaches. A simultaneous modelling with the Markov chain and with the simulator allowed us to validate the simulation tool in order to use it for more realistic network parameters.

According to our experience, the SAN method can be efficiently applied to generate transition matrices (continuous time). Even for advanced computer architectures such as an optical network, a modeller can define synchronisation transitions with "common sense" reasoning and a conception of the SAN rests limpid and clear.

Although the SAN is defined also for discrete time, we have not used it in our modelling practice. In this case it is less obvious to isolate synchronization events in natural way. Moreover, a complex network model requires a lot of them. Taking under consideration the fact that stochastic matrices are "normally" more dense than transition matrices, a task of the non-zero element generation becomes impossible to perform efficiently ([8]).

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