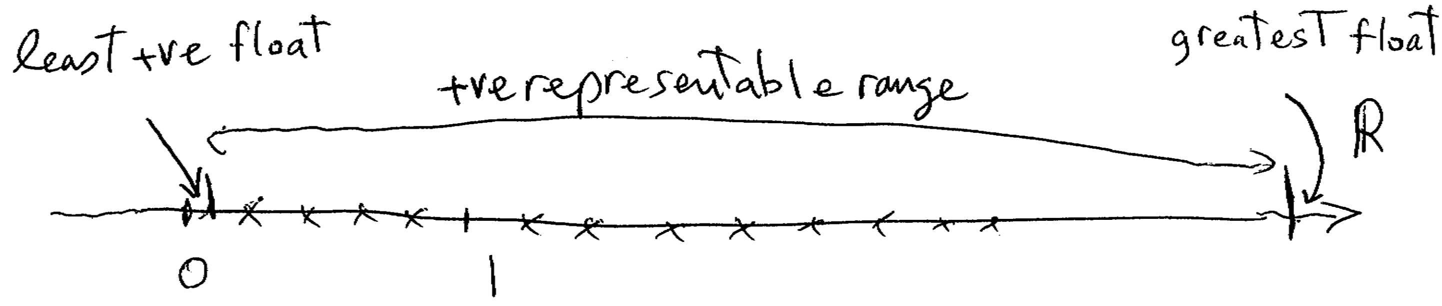
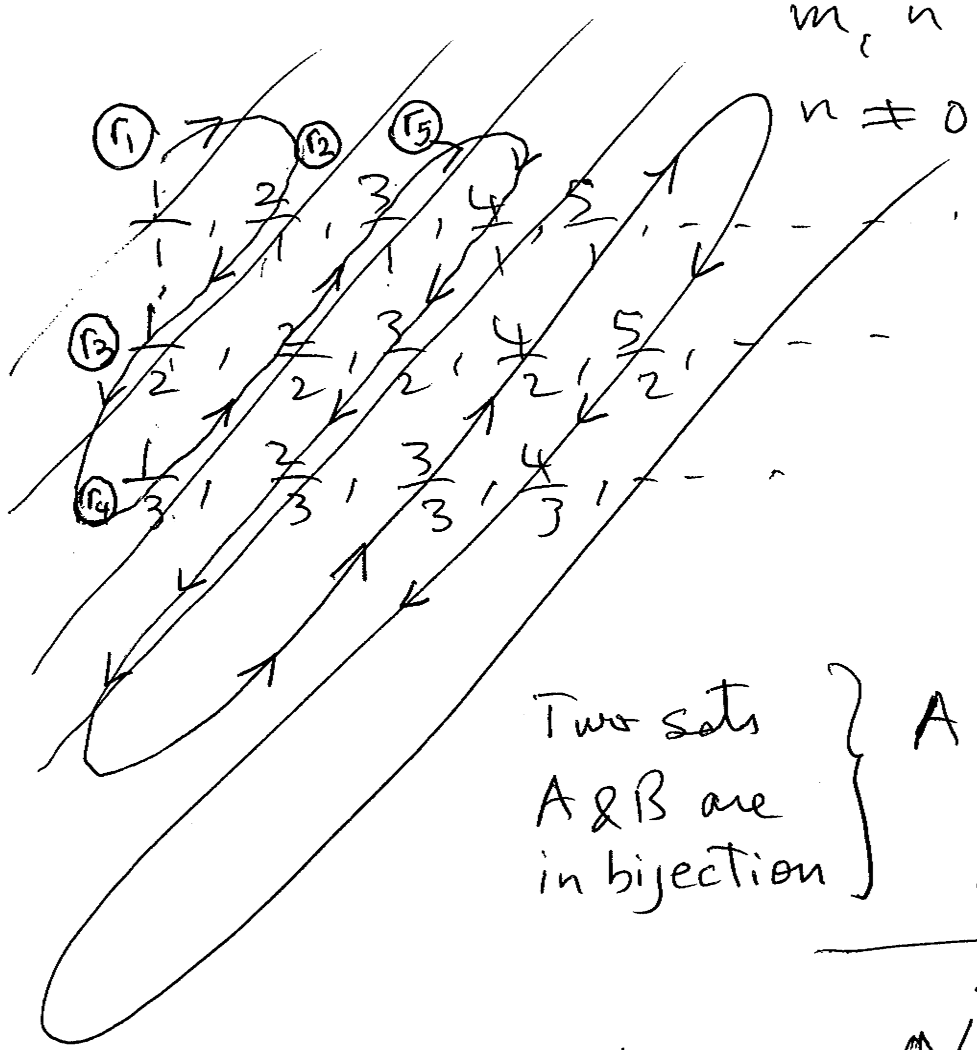


$r \in \mathbb{R}$ is rational if $r = \dfrac{m}{n}$

m, n are integers

$n \neq 0$



Rationals are countable, i.e. they are in a bijective relation with integers.

Two sets A & B are in bijection $\left.\right\}$ $A \overset{g}{\underset{f}{\rightleftarrows}} B$

$f \circ g = g \circ f = \mathrm{Id}$

$\mathbb{N}^{+} \overset{f}{\underset{g}{\rightleftarrows}} \mathbb{Q}^{+}$

+ve integers   +ve rationals

$\boxed{\begin{array}{l}\mathbb{Q} = \text{Rationals} \\ \text{is countable:} \\ r_1, r_2, r_3, \ldots \\ \text{as in the diagram}\end{array}}$

$\sqrt{2}$ is not a rational $\in \mathbb{Q}$

$\pi = 3.14 \cdots$ is not rational

$x^2 = 2$ has root $\pm\sqrt{2}$

$r \in \mathbb{R}$ is algebraic if r is a solution of a polynomial equation with integer coefficients.

$ax + b = 0$

$ax^2 + bx + c = 0$

$ax^3 + bx^2 + cx + d = 0$

We can count algebraic numbers by counting roots of all integer polynomials.

Any number in $[0, 1]$ is of the form

$0.1011101\cdots\cdots$

in base 2

$(0.1\bar{0} = 0.\cancel{0\bar{1}})$

exclude infinite successive occurance of 1 to have unique representation in the binary system.

Algebraic numbers (roots of integer polynomials) are countable

$\boxed{P2}$

Suppose $[0,1]$ is countable. Then write them in a sequence $r_1, r_2, r_3, \ldots$ each in its binary expansion which is unique if we exclude infinite successive 1's.

$r_1 = 0.10101\ldots$

$r_2 = 0.00010\ldots$

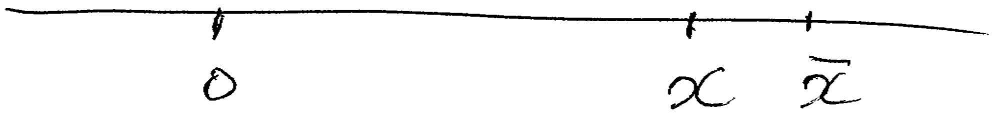
$r_3 = 0.11100\ldots$

Then I construct the number $\hat{r}$:
The n'th digit of $\hat{r}$ is the negation of the n'th digit of r_n: $(\hat{r})_n \neq (r_n)_n$

$\hat{r} = 0.010\ldots$

which is different from all those in the 2-dim array
contradicts the assumption that $[0,1]$ is ~~countable~~ a contradiction.

Therefor: Although rationals and algebraic numbers are each countable, real numbers as a whole (i.e. transcendental numbers) are not countable.



$e_A = |x - \bar{x}|$ absolute error

Definition.

The number of significant digits (base 10) in approximation of x by $\bar{x}$ is: $-\lfloor \log_{10} e_R \rfloor$

- example 1: $-\lfloor \log_{10} 3.18 \times 10^{-2} \rfloor = 2$
- example 2: $-\lfloor \log_{10} 10^{-8} \rfloor = 8$

These are examples in Section 2.1 of the notes.

$$r = \pm m\, b^{z}$$

$$= \pm (bm)\, b^{z-1}$$

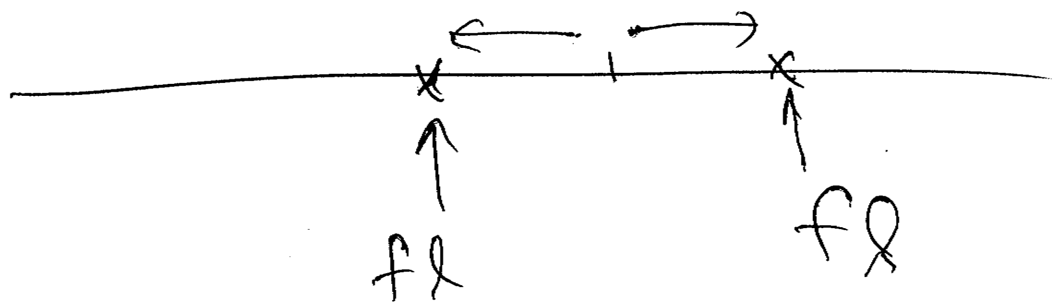
no uniqueness

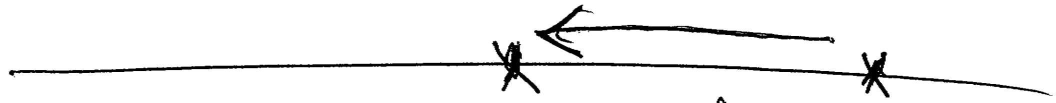
to enforce uniqueness

we assume

$$b^{-1} \le m < 1$$

rounded arithmetic





Truncated arithmetic

$\boxed{\text{Floating point format}}$

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

$$\simeq \log_e n = \ln n$$

Harmonic series grows as $\ln n$ for large n.