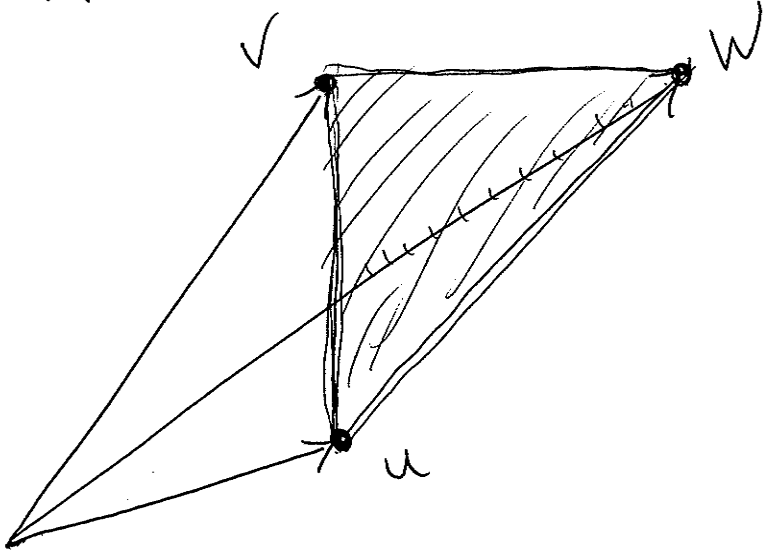
CT notes 10-01-08

Convex linear combination of three vectors in $\mathbb{R}^3$.



$$S = \lambda_1 u + \lambda_2 v + \lambda_3 w$$

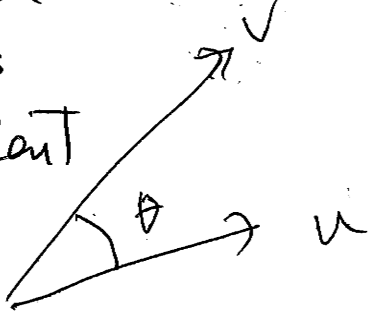
$$\lambda_1, \lambda_2, \lambda_3 \geq 0 \qquad \lambda_1 + \lambda_2 + \lambda_3 = 1$$

$$u \in \mathbb{R}^m, \quad v \in \mathbb{R}^m$$

$$u \cdot v = \sum_{i=1}^{m} u_i v_i$$

$$= |u| \times |v| \cos \theta_{uv}$$

Therefore the scalar product of vectors in $\mathbb{R}^m$ is independant of the coordinate system.



$$u \cdot v = u^T v$$

$$u = \begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix}, \quad v = \begin{pmatrix} v_1 \\ \vdots \\ v_m \end{pmatrix}$$

$$u^T = (u_1, \cdots, u_m)$$

$$u^T v = (u_1, \cdots, u_m) \begin{pmatrix} v_1 \\ \vdots \\ v_m \end{pmatrix}$$

$$= \sum_{i=1}^{m} u_i v_i = u \cdot v$$

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claim:

$$u^T (v_1 + v_2) = u^T v_1 + u^T v_2$$

$$u^T (rv) = r u^T v$$

$r \in \mathbb{R}$

So,

$u^T : \mathbb{R}^m \longrightarrow \mathbb{R}$

u^T is a "linear map"

Def $L : \mathbb{R}^m \to \mathbb{R}$ is linear

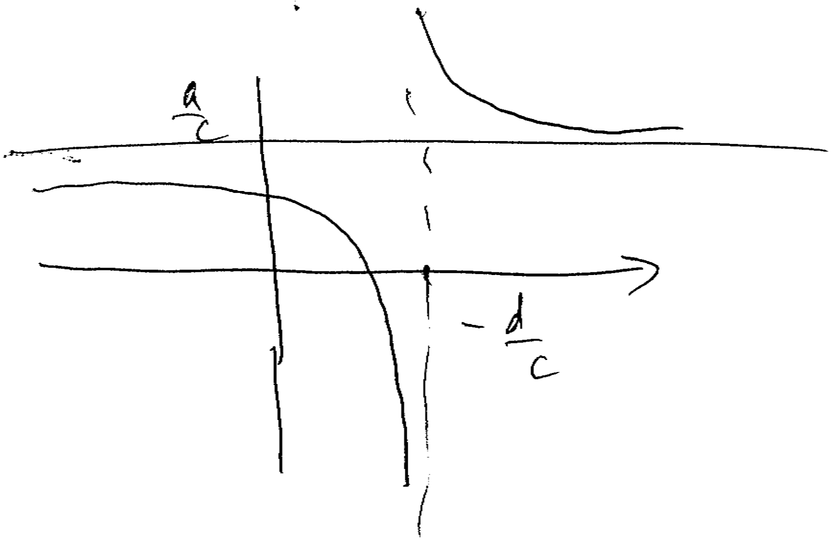
if $L(r_1\vec{v}_1 + r_2\vec{v}_2) =$

$r_1 L(\vec{v}_1) + r_2 L(\vec{v}_2)$

Therefore u^T is really a linear map

All linear maps $\mathbb{R}^m \to \mathbb{R}$ arise as u^T for some $u \in \mathbb{R}^m$.

$$x \longmapsto \frac{ax+b}{cx+d} : \mathbb{R}^\infty \to \mathbb{R}^\infty$$



Note: $L : \mathbb{R} \to \mathbb{R}$ is linear

iff $x \longmapsto rx$ for some $r \in \mathbb{R}$

$$f: \quad x \longmapsto \frac{ax+b}{cx+d}$$

LFT
Linear fractional transformation

$$g: \quad x \longmapsto \frac{a'x+b'}{c'x+d'}$$

$$g \circ f: \quad x \longmapsto \frac{a''x+b''}{c''x+d''}$$

represented by

$$f \sim \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad g \sim \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}$$

represented by

represented by

$$g \circ f \sim \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a'' & b'' \\ c'' & d'' \end{pmatrix}$$

where $\begin{pmatrix} a'' & b'' \\ c'' & d'' \end{pmatrix}$ is simply the matrix product of $\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}$ & $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

This is how matrix multiplication was first used, i.e. to find representation of composition of LFT's.