A diagonal non-square matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{pmatrix}$$

with a change of the orthonormal base (swapping x & y axis) the same matrix is transformed to

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

with singular values $\sigma_1 = 2 \quad \sigma_2 = 1$

For any matrix A, the matrix $A^T A$ is symmetric:

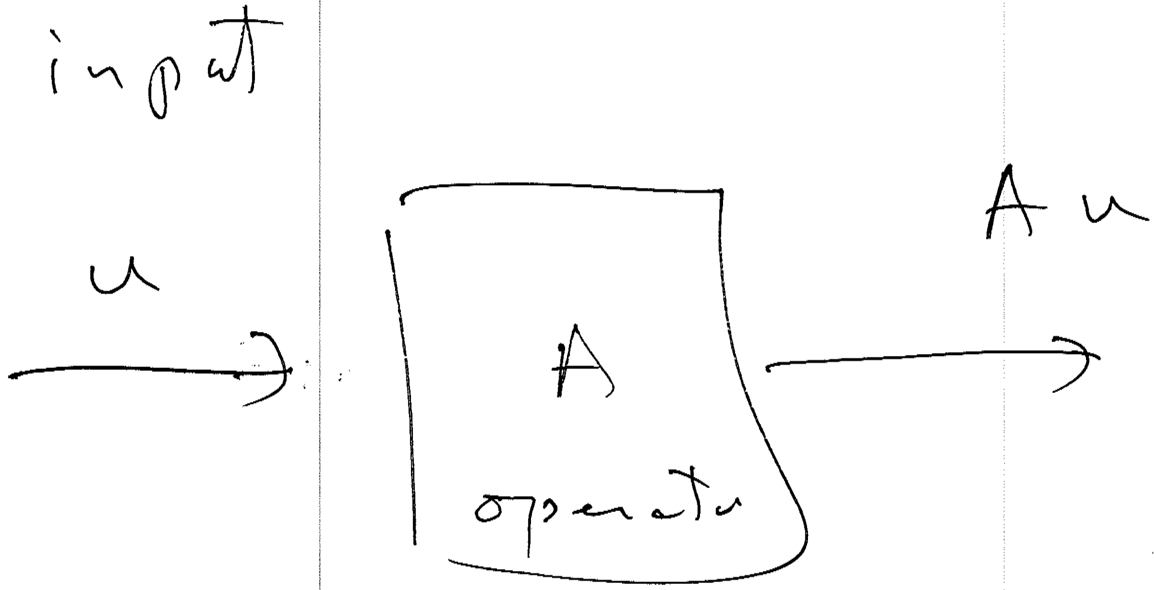
$$(A^T A)^T = A^T (A^T)^T = A^T A$$

$$x^T A x = y^T \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} y = \qquad y = Q^{-1} x$$

$$(y_1, y_2, y_3) \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$= (\lambda_1 y_1 \quad \lambda_2 y_2 \quad \lambda_3 y_3) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$= \underbrace{\lambda_1 y_1^2 + \lambda_2 y_2^2 + \lambda_3 y_3^2 > 0}_{\text{iff}} \quad \lambda_1, \lambda_2, \lambda_3 > 0$$



$$\|Au\|_{\ell_2} \leq \|A\|_{\ell_2} \|u\|_{\ell_2}$$

$Ax = b$

If $A = LU$ where L is lower Triangular &
U is upper "

$LUx = b$ Let $y = Ux$ Then $Ly = b$
obtain y. Then $Ux = y$ so solve for x.

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad \lambda = e^{\pm i\theta}$$

$$A = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \qquad A - \lambda I = \begin{pmatrix} a-\lambda & c \\ b & d-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = (a-\lambda)(d-\lambda) - bc$$

$$= \lambda^2 - \underbrace{(a+d)}_{\text{Trace } A}\lambda + \underbrace{ad - bc}_{\det A} = 0$$

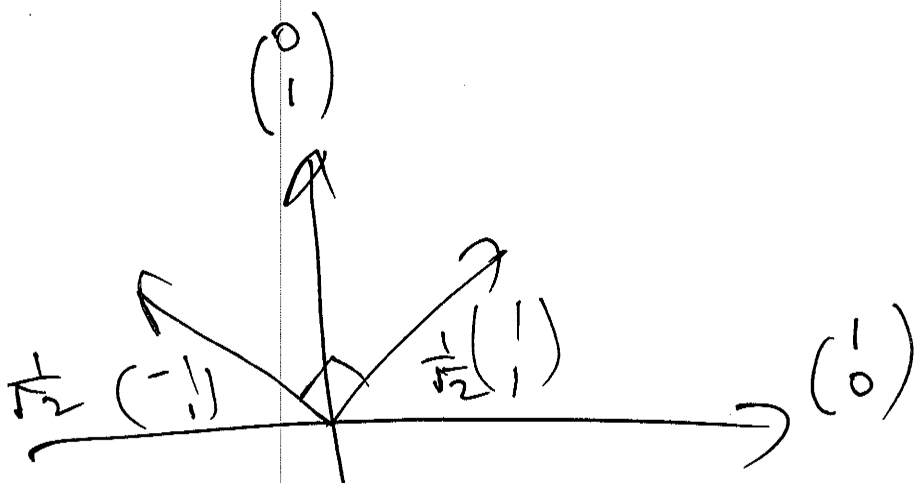
If λ_1 & λ_2 are the roots:

$$\lambda_1 + \lambda_2 = a + d$$

$$\lambda_1 \lambda_2 = ad - bc$$

$$(\lambda - \lambda_1)(\lambda - \lambda_2) = 0$$

$$\lambda^2 - \underline{(\lambda_1 + \lambda_2)}\lambda + \underline{\lambda_1 \lambda_2} = 0$$



$$\left\{ \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix} \right\}$$

An orthonormal base