(i) $\begin{cases} x_1 + x_2 - x_3 = 2 \\ 2x_1 - x_2 + 4x_3 = 4 \\ 3x_1 - 2x_2 - x_3 = -2 \end{cases}$

$Ax = b \qquad A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 4 \\ 3 & -2 & -1 \end{pmatrix}$

$b = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$

$$A^* = \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 2 & -1 & 4 & 4 \\ 3 & -2 & -1 & -2 \end{array}\right) \text{ (i)}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -3 & 6 & 0 \\ 3 & -2 & -1 & -2 \end{array}\right) \text{ (ii)}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -3 & 6 & 0 \\ 0 & -5 & 2 & -8 \end{array}\right) \text{ (iii)}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -3 & 6 & 0 \\ 0 & 0 & -8 & -8 \end{array}\right) \text{ (iv)}$$

(ii)
$$\begin{aligned} x_1 + x_2 - x_3 &= 2 \\ 0x_1 - 3x_2 + 6x_3 &= 0 \\ 3x_1 - 2x_2 - x_3 &= -2 \end{aligned}$$

(iii)
$$\begin{aligned} x_1 + x_2 - x_3 &= 2 \\ 0x_1 - 3x_2 + 6x_3 &= 0 \\ 0x_1 - 5x_2 + 2x_3 &= 4 \end{aligned}$$

$\rightarrow$ (iv)
$$\begin{aligned} x_1 + x_2 - x_3 &= 2 \\ -3x_2 + 6x_3 &= 0 \\ -8x_3 &= -8 \end{aligned}$$

$$x_1 - x_2 + x_3 + 5x_4 = 1$$

$$+3x_2 + 4x_3 + 6x_4 = 2$$

$$0x_3 + 0x_4 = a$$

$$0x_4 = b$$

If $a \neq 0$ or $b \neq 0$, no solution exists.

If $a = b = 0$ Then $\underline{x_3 \text{ and } x_4 \text{ can Take arbitrary values}}$: Solution set $= 2$ dim plane

If $k \neq 0$

x_3 and x_4 can Take arbitrary values

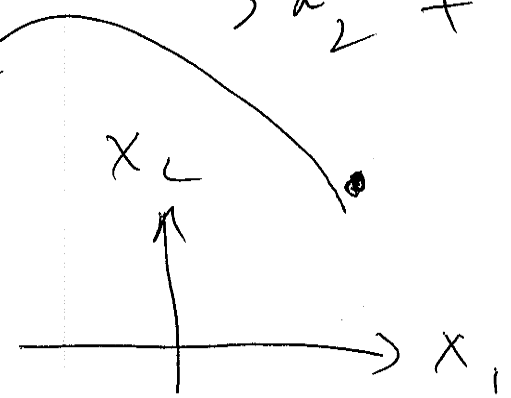
and $\begin{cases} x_1 = \frac{5}{3} \\ x_2 = \frac{6}{5} \end{cases}$

$$3x_1 + 0 + 0 - - + = 5$$

$$5x_2 + 0 \cdots 0 = 6$$

$$0x_3 + 0 \; 0 = k$$

$$0x_4 = 0$$



If $k \neq 0$ no solution exists.

Property (1)

$$\overset{\substack{j^{th}\\ \uparrow}}{(0 \cdots, 1, 0\,0 \cdot)} \begin{pmatrix} a_{11} & a_{12} & a_{1j} & - & - & a_{1n} \\ \vdots & & & & & \\ a_{j1} & - & - & a_{jj} & - & - \\ - & - & - & - & - & - \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix} \to j^{th}$$

$$(a_{j1}, a_{j2}, \cdots, a_{jj}, a_{jj+1} \cdots a_{jn}) \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ 0 \end{pmatrix} \to j^{th} \text{ entry}$$

$$= a_{jj} > 0$$

This shows that if A is positive definite then all its diagonal entries are positive.

Property (2): The largest in magnitude of entries in the matrix will be in the diagonal

$$(1, -1, 0) \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = a + d - 2b < 0$$

For example to show that b is not the largest element use $x = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ which gives a contradiction

Property (4)

A symmetric $n \times n$

$$x^T A x = y^T \Lambda y = \sum_{i=1}^{n} \lambda_i y_i^2$$

$$\Lambda = \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \ \lambda_n \end{pmatrix}$$

$$y = Q^T x$$

$$Q = [v_1, \ldots v_n]$$

eigenvectors of A

Property (5)

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & - & - \\ a_{21} & a_{22} & a_{23} & - & - \\ a_{31} & a_{31} & a_{33} & - & - \\ - & - & - & - & \end{pmatrix}$$

If A is +ve definite then all its principle minors are also positive definite.

$$A = \begin{pmatrix} a & b & c \\ b & e & f \\ c & f & g \end{pmatrix}$$

A positive definite $\Rightarrow \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ is
(i.e. $x^T A x > 0$) also positive definite

Choose $x = \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix}$ then $x^T A x = (x_1, x_2) \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} > 0$