A symmetric

$$x^T A x = y^T \Lambda y = \sum \lambda_i y_i^2 > 0$$

iff $\lambda_i > 0$ $(\forall i = 1 \sim n)$

$$\Lambda = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$$

$$\boxed{y = Q^T x}$$

---

if $A = LL^T$ then A is positive definite

i.e. $x^T A x > 0$ $\forall x \in \mathbb{R}^n \setminus \{0\}$

__Proof__

Let $x \in \mathbb{R}^n$ then

$$x^T A x = x^T L L^T x = (L^T x)^T L^T x = y^T y \geq 0$$

$(L^T x)^T = x^T (L^T)^T = x^T L$

where $y = L^T x$

$Ax = b$ — If $A = LU$

L: lower triangular

U: upper triangular

$L(Ux) = b$

Let $y = Ux$

$Ly = b$ — find y by forward ~~back~~ substitution

$Ux = y$ — when y is now ~~not~~ known, solve for x using back substitution

$$\begin{cases} x_1 + x_2 + x_3 = 4 \\ x_2 - x_3 = 5 \\ 0x_3 = a \end{cases}$$

(i) if $a \neq 0$ then no solution exists

(ii) if $a = 0$ then

$$x_2 = x_3 + 5$$

and $x_1 = -x_2 - x_3 + 4 = -x_3 - 5 - x_3 + 4$

$$= -2x_3 - 1$$

$$\begin{cases} x_1 = -2\alpha - 1 \\ x_2 = \alpha + 5 \\ x_3 = \alpha \end{cases}$$