Example 23, 25, 27 $\quad A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{pmatrix}$

$\cancel{A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{pmatrix}}$, $\quad \text{range}(A) = \begin{pmatrix} \beta_1 + \beta_2 \\ \beta_1 \\ \beta_1 - \beta_2 \end{pmatrix}$, $\quad \text{Null}(A^T) = \begin{pmatrix} \beta_3 \\ -2\beta_3 \\ \beta_3 \end{pmatrix}$

with $\beta_1, \beta_2 \in \mathbb{R}$ $\qquad$ with $\beta_3 \in \mathbb{R}$

$$Ax = b = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

$\begin{cases} \beta_1 = 0 \\ \beta_2 = 1 \end{cases} \quad \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \in \text{range}(A)$

$\beta_3 = 1 \quad \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \in \text{Null}(A^T)$

$b = b_R + b_N$

$b \in \text{Range}(A)$ iff $b_N = 0$

$b \in \text{Null}(A^T)$ iff $b_R = 0$

$$\begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$Ax = b_R = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

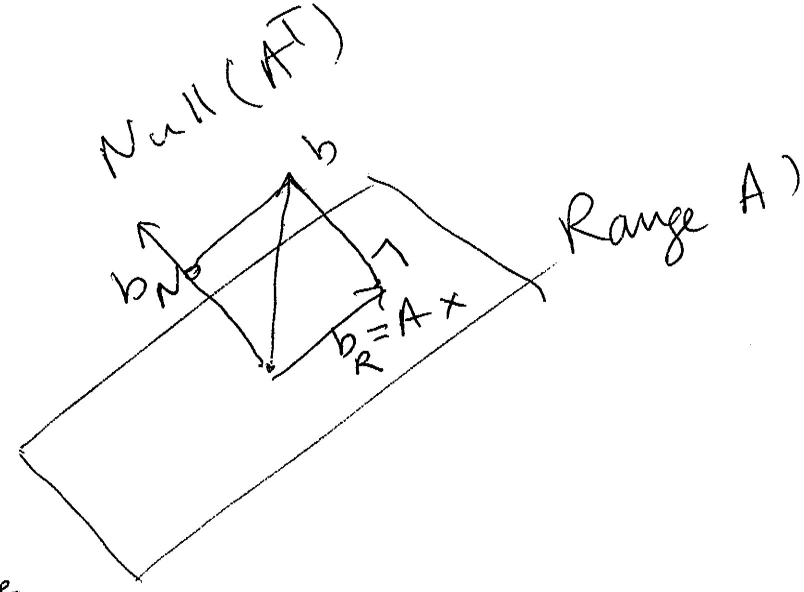
$x_1 + x_2 = 1$

$x_1 = 0, \quad x_1 - x_2 = -1$

$\boxed{\begin{matrix} x_1 = 0 \\ x_2 = 1 \end{matrix}}$

$\| b - Ax \|_2$

A $m \times n$



$b \notin$ Range(A)

otherwise for some

x $Ax = b$

Given b, b_R & b_N are unique

$b = b_N + b_R$ $\quad b_N \in$ Null(A^T), $b_R \in$ Range(A)

So solve for $Ax = b_R$ where

This will always have a solution.

If the columns of A are linearly independant then $Ax = b_R$ has a unique solution

[otherwise if $Ay = b_R$ then we set $A(x-y) = 0$

$\Rightarrow x - y = 0$

In many areas of practical application we have to solve

$$Ax = b_i \quad \text{for } i = 1, \ldots, N$$

with large N

For efficiency we proceed as follows:

Assume $Ax = b$ has no exact solution; find $b = b_R + b_N$

Solve $Ax = b_R$

$$Ax = b = b_R + b_N \Rightarrow Ax - b_R = b_N$$

$$A^T(Ax - b_R) = A^T b_N = 0$$

$$\boxed{A^T A x = A^T b_R = A^T b} \quad (\text{since } A^T b_N = 0)$$

Find L such that $A^T A = L L^T$ and use the technique of solving $L L^T x = y$ to find all solutions.

$$y = A^T b$$

$y = f(x)$ If x_0 minimizes $f(x)$

then x_0 minimizes

$y^2 = (f(x))^2$

$2(f(x))(f'(x)) = 0 \Rightarrow f'(x) = 0$