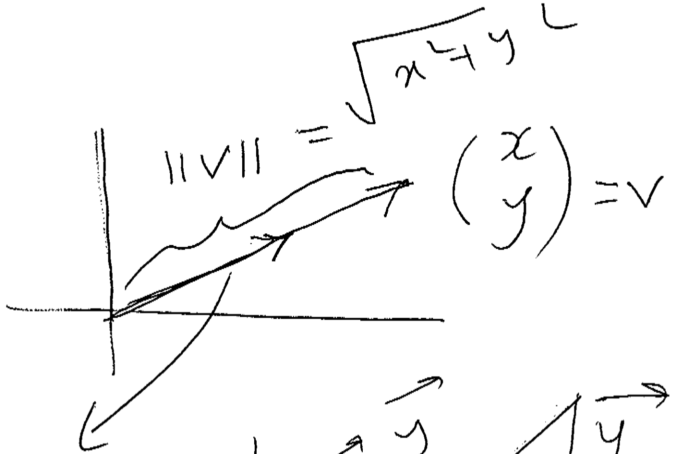


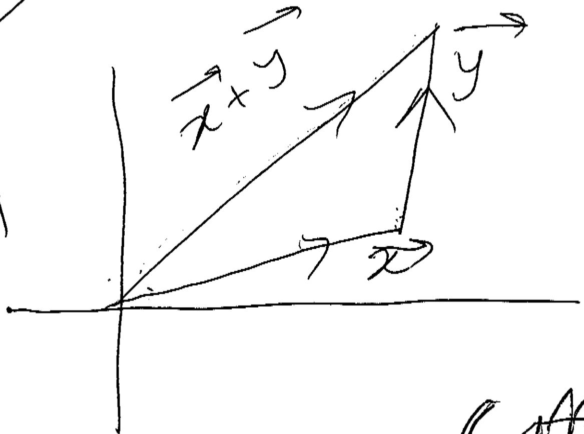
$\|v\| = \sqrt{x^2+y^2}$

$v \neq 0$

$\left\| \frac{v}{\|v\|} \right\|$

$= \frac{1}{\|v\|} \|v\|$

$= 1$

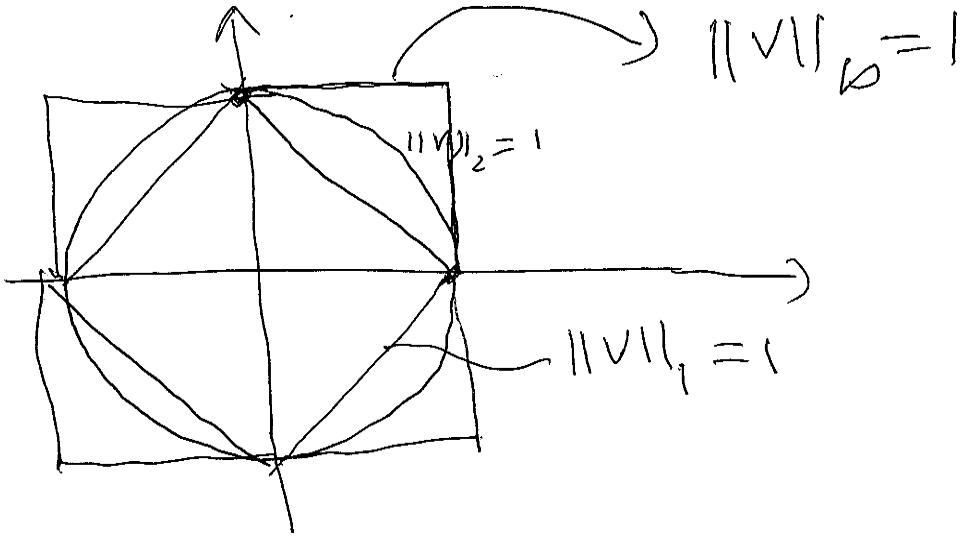


$$\|x\|_p = \left(\sum_{i=1}^{m} |x_i|^p \right)^{\frac{1}{p}}$$

$$= \max_{1\le i\le m} |x_i| \left(\sum_{1\le i\le n} \left(\frac{|x_i|}{\max |x_i|} \right)^p \right)^{\frac{1}{p}}$$

$\cancel{(x)^p \quad \sqrt[p]{x}}$

ℓ^2 norm $\|v\|_2 = 1$ $v = \begin{pmatrix} x \\ y \end{pmatrix}$



ℓ^1 norm $\|v\|_1 = 1$ $\|v\|_1 = |x| + |y| = 1$

if $x \geq 0, y \geq 0$ $x + y = 1$

ℓ^∞ norm $\|v\|_\infty = 1$ $\|v\|_\infty = \max\{|x|, |y|\}$
$= 1$

Assume $x \geq 0, y \geq 0$
$\|v\|_\infty = \max\{x, y\} = 1$

$$u = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} \qquad v = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$$

$$\sqrt{u_1^2 + \cdots u_n^2}\sqrt{v_1^2 + \cdots + v_m^2} \geq \sum u_i v_i = u_1 v_1 + \cdots u_n v_n$$

$\lambda \in \mathbb{R}$

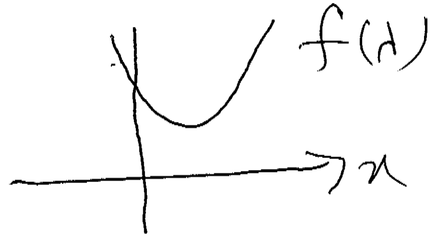
real number

$$W = u - \lambda v$$

$$0 \leq \|W\| = \sqrt{(u - \lambda v)^T (u - \lambda v)}$$

$$= \sqrt{(u^T - \lambda v^T)(u - \lambda v)}$$

$$= \sqrt{u^T u - \lambda \underline{u^T v} - \lambda \underline{v^T u} + \lambda^2 v^T v}$$



$$f(\lambda) = \|u\|^2 - 2\lambda(u \cdot v) + \lambda^2 \|v\|^2 \geq 0$$

$$\Longrightarrow (u \cdot v)^2 - \|u\|^2 \|v\|^2 \leq 0$$