Quantum Computing

Assessed Coursework (II)

1. In the Grover's algorithm, let $n = 2$ so that $N = 2^n = 4$, and let $M = 1$. Show that the oracle f with $f(x) = 0$ for all $x \neq x_0$ and $f(x_0) = 1$ can be chosen from the four circuits in Figure 1.

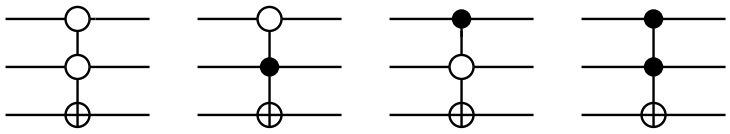

Figure 1: Four possible oracles

Show that the circuit in Figure 2 in effect implements the operation G. How many iterates of G are needed to determine x_0?

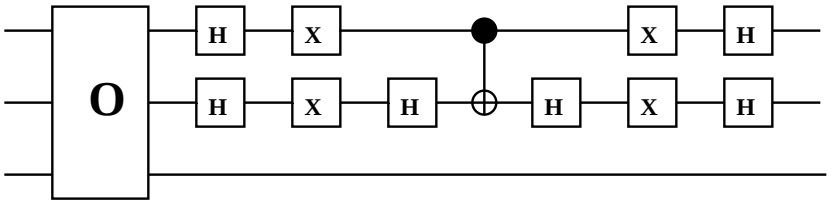

Figure 2: Circuit for G

2. In the phase estimation algorithm, suppose the states $|u\rangle$ for $u \in T$ are eigenstates of U with eigenvalue $e^{2\pi i\phi_u}$. The phase estimation algorithm maps the normalized state

$$|0\rangle(\sum_{u\in T} d_u|u\rangle)$$

to the state

$$\sum_{u\in T} d_u|\hat{\phi}_u\rangle|u\rangle,$$

where the state $|\hat{\phi}_u\rangle$ gives a good estimate of ϕ_u. Show that with t chosen as in Equation 18, page 100 of the notes, the probability of measuring ϕ_u accurate to s bits in the output of the phase estimation algorithm is at least $|d_u|^2(1-\epsilon)$.