# Domains and Differential Calculus: Functions of $n$ Variables

15th December 2003

## 1 Partial Interval Derivatives

**Notation 1.1.** If $n \geq 1$, the canonical projection $\mathbb{R}^n \to \mathbb{R}$, $(x_1, \ldots, x_n) \mapsto x_i$ is denoted by $\pi_i$. The function which delets the $i$-th component is denoted by $\Pi_i : \mathbb{R}^n \to \mathbb{R}^{n-1}$, $(x_1, \ldots, x_n) \mapsto (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$.

**Definition 1.2 (Partial Interval Derivatives).** Suppose $f : \mathrm{I}[0,1]^n \to \mathbb{IR}$ and $a \in \mathrm{I}[0,1]^n$ and $b \in \mathbb{IR}$.

1. For $b \in \mathbb{IR}$ put

$$f \in \delta_i(a, b) \quad \text{iff} \quad b(\pi_i(x_1) - \pi_i(x_2)) \sqsubseteq f(x_1) - f(x_2).$$

   for all $x_1, x_2 \ll a$ with $\Pi_i(x) = \Pi_i(x_2)$.

2. For $b = (b_1, \ldots, b_n) \in \mathbb{IR}^n$, we let $\delta(a, b) = \bigcap_{1 \leq i \leq n} \delta_i(a, b_i)$. In this case, we say that $f$ *has interval derivative $b$ at $a$.*