Dynamical Systems and Deep Learning - Exercises 1

Exercises marked with * are more challenging and are designed for students with more mathematical background.

1. Find the dominant term and the smallest big O complexity of the following expressions as $x \to \infty$:

- $98x \log x - 23x^{1.1}$.
- $7x^2 - \frac{4x^3}{\log x}$.

2. Decide whether each statement below is true or false as $x \to \infty$ and prove your assertion:

- $(x^5 + 12x^4 - 3x + 2)/(3 + x^5) = o(1)$.
- $3x^2 - 4x + 5 \sim x^2$.
- $x^{-1.2} - 5x^{-2} \sim x^{-1.2}$.
- $f_1 = O(g_1)$ and $f_2 = O(g_2) \;\Rightarrow\; f_1 f_2 = O(g_1 g_2)$.
- $f_1 = o(g)$ and $f_2 = o(g) \;\Rightarrow\; f_1 + f_2 = o(g)$ and $f_1 f_2 = o(g)$.

3. Determine the type of the fixed points of the map $F : \mathbb{R} \to \mathbb{R}$ with $F(x) = x(1 - x)$ and sketch its phase portrait.

4. Find all fixed points of $F : \mathbb{R} \to \mathbb{R}$ with $F(x) = x^3 - \frac{7}{9}x$ and determine whether they are attracting, repelling or neither. Sketch the phase portrait of the map.

5. Find all fixed points of $F : \mathbb{R} \to \mathbb{R}$ with $F(x) = x^3 - 2x$ and determine their nature. Show that F has a period orbit $\{1, -1\}$ of period 2. Sketch the graph of F and thus determine the type of the periodic orbit $\{1, -1\}$.
(*) Use Taylor series expansion to deduce the type of this periodic orbit.