1. Find all fixed points of $F : \mathbb{R} \to \mathbb{R}$ with $F(x) = x^3 - 2x$ and determine their nature. Show that $F$ has a period orbit $\{1, -1\}$ of period 2. What is the type of this periodic orbit?

2. Find the explicit form of
   
   (i) the maps $f_1, f_2, f_3$ in the generation of the Sierpinsky triangle, and
   
   (ii) the maps $f_1, f_2, f_3, f_4$ in the generation of the Koch curve.

3. Find the fixed points of each of the maps $f_1, f_2, f_3, f_4$ in the generation of the Koch curve. What points of $\{1, 2, 3, 4\}^\mathbb{N}$ correspond to these points? Do the same for the Sierpinski triangle and the Cantor set.

4. Consider the Cantor set $C$ and its generating sequence $\langle I_n \rangle_{n \geq 0}$. Find $d_H(I_n, C)$ in $\mathcal{P}(\mathbb{R})$.

5. Find the attractor of the IFS $\{f_1, f_2\}$ in $\mathbb{R}$, where, for $x \in \mathbb{R}$, $f_1(x) = ax$ and $f_2(x) = (1 - a)x + a$, with $0 < a < 1$.

6. Repeat question 5 with $f_1, f_2$ given by $f_1(x) = 0$ and $f_2(x) = \frac{2}{3}x + \frac{1}{3}$.

7. Describe the attractor of the IFS, $f_0, f_1, f_2 : \mathbb{R} \to \mathbb{R}$ with $f_j \mapsto \frac{x}{4} + \frac{3j}{8}$ ($j = 1, 2, 3$) and find its similarity dimension.
8.

(i) Show that if \( g : \mathbb{R}^m \to \mathbb{R}^m \) has contracting factor \( s < 1 \), then the closed ball with centre \( u \in \mathbb{R}^m \) and of radius \( \|u - g(u)\|/(1 - s) \) is a trapping region for \( g \), i.e., is mapped by \( g \) into itself.

(ii) Given an IFS, \( f_1, f_2, \ldots, f_N : \mathbb{R}^m \to \mathbb{R}^m \), find a closed ball centred at \( u \) which is mapped by \( f \) into itself, where \( f : \mathcal{P}(\mathbb{R}^m) \to \mathcal{P}(\mathbb{R}^m) : A \mapsto \bigcup_{i=1}^N f_i[A] \).

(iii)* When \( m = 2 \) and \( N = 2 \) and the maps \( f_i : \mathbb{R}^2 \to \mathbb{R}^2 \) (\( 1 \leq i \leq 2 \)) are both affine, explain how you would find \( u \in \mathbb{R}^2 \) and \( R \geq 0 \) such that the closed ball \( C(u, R) \) is the smallest trapping disk for \( f \)?

9*. Show that if \( F : [a, b] \to \mathbb{R} \) is differentiable and its derivative \( F' \) is continuous at a fixed point \( x_0 \) of \( F \), then \( x_0 \) is attracting (repelling) if \( |F'(x_0)| < 1 \) (\( |F'(x_0)| > 1 \)).

Hint: Assume \( |F'(x_0)| < 1 \). The continuity of \( F' \) at \( x_0 \) implies that there exists some \( \delta > 0 \) such that \( |F'(x)| < k \) for \( x_0 - \delta < x < x_0 + \delta \) where \( k = (|F'(x_0)| + 1)/2 \) (simply put \( \epsilon = (1 - |F'(x_0)|)/2 \) in the definition of continuity of \( F' \) at \( x_0 \)). Now apply the mean value theorem to \([x_0, x]\) or \([x, x_0]\) where \( |x_0 - x| < \delta \) and note that \( k < 1 \).