1. Show that the clustering coefficient for a one dimensional lattice with periodic boundary condition (i.e., a circle), as for example in the figure below, can be computed to be

\[ C = \frac{3(z - 2)}{4(z - 1)} \]

which tends to \(3/4\) as \(z \to \infty\). (Here, \(z \ll N\) and \(z\) is assumed to be even so that every vertex has \(z/2\) connections with its neighbours on one side and \(z/2\) connections on the other side.)

![Diagram of a one-dimensional lattice with periodic boundary condition](image)

2. Find the average distance in a one dimensional lattice of length \(\ell\) with \(z = 2\) and obtain its asymptotic behaviour as \(\ell \to \infty\).

3. We can equivalently define a random graph by its size \(N\) and its total number of edges \(n\).

   (i) What is the total number of possible graphs with this specification?

   (ii) Find \(z\) and \(p\) (as defined in the notes) in terms of \(N\) and \(n\).

   (iii) Starting with the definition of a random network as in the notes, find the expected value \(\langle n \rangle\) of the number of edges \(n\).
4. Find the expected value and the second moment of the degree of vertices

\[ \langle k \rangle = \sum_{k=1}^{\infty} k P(k), \]
\[ \langle k^2 \rangle = \sum_{k=1}^{\infty} k^2 P(k), \]

for the random growing network, where \( P(k) = 2^{-k} \). Hence, find \( z_2/z_1 \) and find the average number of nodes \( n \) steps away from a vertex.

**Hint:** Evaluate \( \langle k \rangle = 2\langle k \rangle - \langle \rangle \) and \( \langle k^2 \rangle = 2\langle k^2 \rangle - \langle k^2 \rangle \).