Complex Systems- Exercises 6

1. Assume we have three boolean networks all with $N = 3$, $K = 1$ and a cyclic linkage tree

$$f_1 : \sigma_2 \mapsto \sigma_1, \qquad f_2 : \sigma_3 \mapsto \sigma_2, \qquad f_3 : \sigma_1 \mapsto \sigma_3.$$

The coupling functions f_i ($i = 1, 2, 3$) for the three networks, labelled (i) ,(ii) and (iii), are given as follows:

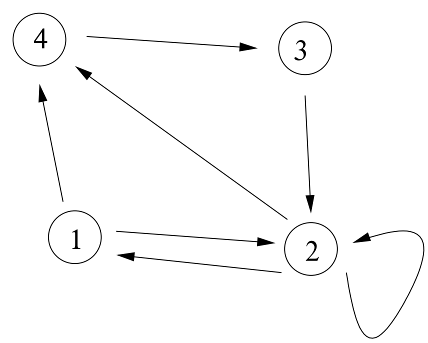
(i) $f_1 = f_2 = f_3 =$ identity.

(ii) $f_1 = f_2 = f_3 =$ negation.

(iii) $f_1 = f_2 =$ negation and $f_3 =$ identity.

For each of the three networks, determine all cycles and their attraction basins.

2. Consider the network with four boolean variables $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ ($N = 4$) below. The boolean variable σ_1 has one controlling variable, namely σ_2, i.e., $K_1 = 1$, while σ_2 has three controlling variables, namely $\sigma_1, \sigma_2, \sigma_3$, i.e., $K_2 = 3$. For others, we have $K_3 = 1$ and $K_4 = 2$.



Assume that the four boolean functions $f_i : \{0, 1\}^{K_i} \to \{0, 1\}$ with $1 \le i \le 4$ are all generalised XOR-functions, i.e., their output is 1, respectively 0, if the number of input 1's is odd, respectively even. (i) Find all cycles of the network. (ii) Let $\Sigma_0 = 0001$ and $\hat{\Sigma}_0 = 1001$. Find the Hamming distance $h(\Sigma_t, \hat{\Sigma}_t)$ for all $t \ge 0$.