Complex Systems- Exercises 6 (solutions)

1. Assume we have three boolean networks all with $N = 3$, $K = 1$ and a cyclic linkage tree

$$f_1 : \sigma_2 \mapsto \sigma_1, \qquad f_2 : \sigma_3 \mapsto \sigma_2, \qquad f_3 : \sigma_1 \mapsto \sigma_3.$$

The coupling functions f_i ($i = 1, 2, 3$) for the three networks, labelled (i) ,(ii) and (iii), are given as follows:

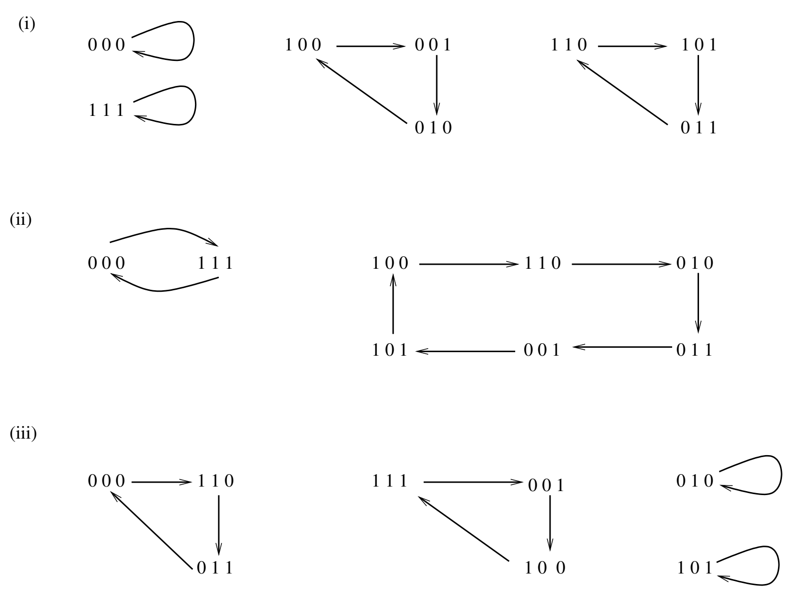
(i) $f_1 = f_2 = f_3 =$ identity.

(ii) $f_1 = f_2 = f_3 =$ negation.

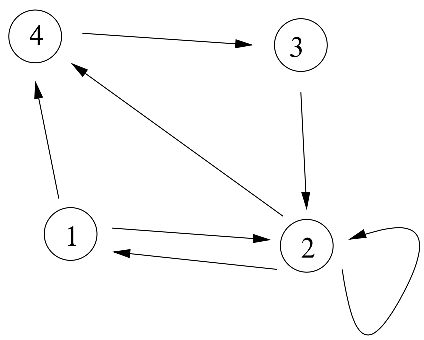
(iii) $f_1 = f_2 =$ negation and $f_3 =$ identity.

For each of the three networks, determine all cycles and their attraction basins.

Solution. The cycles for the three networks are given below. In each case the basin of attraction coincides with the cycle.



2. Consider the network with four boolean variables $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ ($N = 4$) below. The boolean variable σ_1 has one controlling variable, namely σ_2, i.e., $K_1 = 1$, while σ_2 has three controlling variables, namely $\sigma_1, \sigma_2, \sigma_3$, i.e., $K_2 = 3$. For others, we have $K_3 = 1$ and $K_4 = 2$.



Assume that the four boolean functions $f_i : \{0,1\}^{K_i} \to \{0,1\}$ with $1 \le i \le 4$ are all generalised XOR-functions, i.e., their output is 1, respectively 0, if the number of input 1's is odd, respectively even. (i) Find all cycles of the network. (ii) Let $\Sigma_0 = 0001$ and $\hat{\Sigma}_0 = 1001$. Find the Hamming distance $h(\Sigma_t, \hat{\Sigma}_t)$ for all $t \ge 0$.

Solution. (i) There are four cycles as in the diagram below.
(ii)

$$h(\Sigma_t, \hat{\Sigma}_t) = \begin{cases} 1 & t = 0 (\mathrm{mod}\ 5) \\ 2 & t = 1 (\mathrm{mod}\ 5) \\ 4 & t = 2 (\mathrm{mod}\ 5) \\ 3 & t = 3 (\mathrm{mod}\ 5) \\ 2 & t = 4 (\mathrm{mod}\ 5) \end{cases}$$