Complex Systems- Exercises 7

1. (i) When $K = 1$, explain whether or not any module is a loop linkage in the Kauffman boolean network.
(ii) Suppose for $K = 1$ the module diagram in a Kauffman boolean network consists of a single loop. If in the quenched model one of the coupling functions is constant, what can we say about the number and size of the cycles and the size of their basins of attraction?
2. Assume that $K = N$ and that we can compute probabilities for going from one configuration to another as if the system goes through a random walk, as in the notes. The probability of having an open trajectory at time t is q_t with

$$q_{t+1} = q_t(1 - \frac{t+1}{\Omega})$$

as in the notes.

- Show that the probability p_{t+1} of terminating the excursion at time $t+1$ is

$$p_{t+1} = \frac{t+1}{\Omega} q_t.$$

- Show that starting with a given initial state the closure of the excursion at time t provides with equal probability all cycle lengths up to t.

- Show that the probability $P(L)$ that a given initial state is in the basin of attraction of a cycle of length L is given by:

$$P(L) = \sum_{t=L}^{\Omega} \frac{p_t}{t}.$$

- Using the asymptotic approximation $1-x \sim e^{-x}$ as $x \to 0$ and thus $1-\frac{t}{\Omega} \approx e^{-\frac{t}{\Omega}}$ for large Ω, show that for large Ω we have:

$$\frac{p_t}{t} \approx \frac{1}{\Omega} \exp -(\sum_{s=1}^{t-1} \frac{s}{\Omega}).$$

- Hence, show that if Ω is large we have the approximation:

$$P(L) \approx \int_L^\infty \frac{1}{\Omega} \exp(-x(x-1)/2\Omega)\, dx.$$