Quantum Computing - Exercises 3

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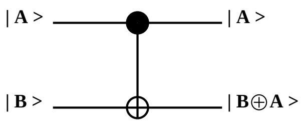
1. Check that the matrix representing CNOT in the standard computational basis is the one given in Figure 1 and that it is unitary.



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Figure 1: The CNOT gate

2. Show that the circuit on the left in Figure 2, swaps the two qubits. It is denoted by the figure on the right. What is its matrix representation?

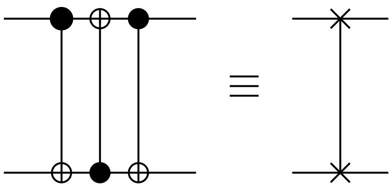


Figure 2: Swapping Gate

3. There is nothing special about the control bit in the CNOT gate to take value one. The control gate in Figure 3 flips the target bit if the control bit is zero. This is represented pictorially by a hollow circle instead of the filled circle on the top line. What is its matrix representation?

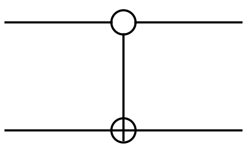


Figure 3: A variation of CNOT

4. Check that the Toffoli gate (Figure 4) transformation is unitary, and thus can be implemented as a quantum gate.

IN			OUT		
a	b	c	a'	b'	c'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

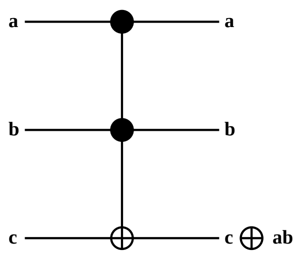


Figure 4: The Toffoli gate

5.

(i) Show the bilinearity property: $(\alpha v + \alpha' v') \otimes (\beta w + \beta' w') =$ $\alpha\beta v \otimes w + \alpha\beta' v \otimes w' + \alpha'\beta v' \otimes w + \alpha'\beta' v' \otimes w'$, where $\alpha, \alpha', \beta, \beta' \in \mathbb{C}$, $v, v' \in \mathbb{C}^k$, $w, w' \in \mathbb{C}^l$.

(ii) Show that $b_i^k \otimes b_j^l = b_{(i-1)l+j}^{kl}$.

(iii) Show that for $v, v' \in \mathbb{C}^k$ and $w, w' \in \mathbb{C}^l$ we have: $\langle v \otimes w | v' \otimes w' \rangle = \langle v | v' \rangle \langle w | w' \rangle$.

6.

(i) Show that:

$$(M \otimes N)(v \otimes w) = (Mv) \otimes (Nw).$$

(ii) Show the bilinearity property:

$$(\alpha M + \alpha' M') \otimes (\beta N + \beta' N') =$$
$$\alpha\beta M \otimes N + \alpha\beta' M \otimes N' + \alpha'\beta M' \otimes N + \alpha'\beta' M' \otimes N'.$$

(iii) Show that:

$$(M \otimes N)(M' \otimes N') = (MM') \otimes (NN').$$

(iv) Show that

$$\begin{aligned} (M \otimes N)^* &= M^* \otimes N^*, \\ (M \otimes N)^T &= M^T \otimes N^T \\ (M \otimes N)^\dagger &= M^\dagger \otimes N^\dagger. \end{aligned}$$

(v) Show that $M \otimes N$ is unitary (invertible) if M and N are unitary (invertible).