Quantum Computing - Exercises 5

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1. Check that the output of the network in Figure 1 is indeed as in the table on its left up to the normalization factor $1/\sqrt{2}$.

in	out
|00>	(|00> + |11>)
|01>	(|01> + |10>)
|10>	(|00> − |11>)
|11>	(|01> − |10>)

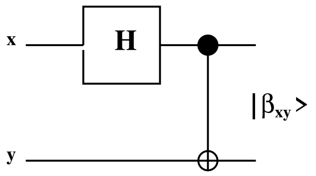

Figure 1: The Bell or EPR states

2. In the quantum teleportation network of Figure 2 (see page 61 in the notes), measurement of the first two qubits by Alice will collapse Bob's qubit as follows:

$$00 \mapsto |\psi_3(00)\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$01 \mapsto |\psi_3(01)\rangle = \alpha|1\rangle + \beta|0\rangle$$

$$10 \mapsto |\psi_3(10)\rangle = \alpha|0\rangle - \beta|1\rangle$$

$$11 \mapsto |\psi_3(11)\rangle = \alpha|1\rangle - \beta|0\rangle$$

Alice communicates her two bits mn with Bob over a classical channel. Bob will then send his qubit through the circuit $X^n Z^m$ where

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Check that the final result $|\psi_4\rangle$ is indeed the state $|\psi_4\rangle = |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

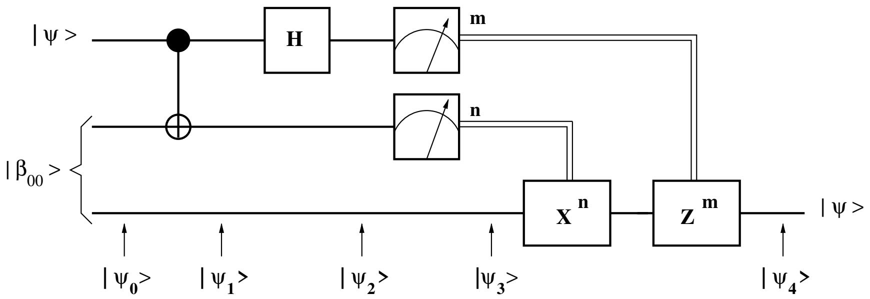

Figure 2: Quantum teleportation

3. Let $f : \{0,1\} \to \{0,1\}$ be a Boolean function which outputs a bit for each input bit and let $U_f : |x, y\rangle \mapsto |x, y \oplus f(x)\rangle$. Show that U_f is unitary.

4. More generally, let $f : \{0,1\}^n \to \{0,1\}$ and $U_f : \mathbb{C}^{n+1} \to \mathbb{C}^{n+1}$ with $U_f : |x, y\rangle \mapsto |x, y \oplus f(x)\rangle$. Now U_f has $n+1$ input qubits, the first n are input to f; it has also $n+1$ output qubits, the first n qubits are identical with the first n qubits of the input as in Figure 3. Check that for any integer n the operator U_f is a unitary transformation.

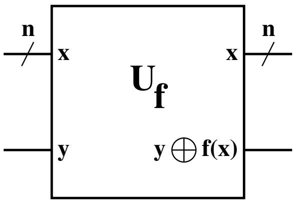

Figure 3: A Gate for Parallel Computation

5. Show that

$$H^{\otimes n}|0\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle.$$

6. Show that

$$\begin{aligned} H|x\rangle &= \textstyle\sum_{y \in \{0,1\}} (-1)^{xy} |y\rangle / \sqrt{2} \\ H^{\otimes n}|x\rangle &= \textstyle\sum_{y \in \{0,1\}^n} (-1)^{x.y} |y\rangle / \sqrt{2^n} \end{aligned}$$

where $x.y$ is the bitwise inner product of x and y modulo 2.