Domain theory and Continuous Data Types
Exercises 5

1. Obtain the real number represented by the continued fraction expansion

$$1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{1 + \ddots}}}$$

Find the first three intervals of approximation.

2. Consider the following lft's:

$$D_{-1} = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} \quad D_0 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \quad D_1 = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}.$$

Find $D_{-1}[0,\infty]$, $D_0[0,\infty]$ and $D_1[0,\infty]$. Show that these maps contract the ρ-distance on $[0,\infty]$ (page 55) uniformly by a factor $1/2$. What numbers are represented by the normal products D_{-1}^{ω}, D_0^{ω} and D_1^{ω}?

3. Check that the rules for left and right absorptions into a tensor correspond to composition of lft's.

4. Obtain the expression tree for $\arctan(\sqrt{2}/2)$ using the unp for $\sqrt{2}$ on page 61. Work out the interval represented by $\arctan(\sqrt{2}/2)$ when the first three matrices for the unp of $\sqrt{2}/2$ are absorbed into the first two tensors of the expression for $\arctan x$ (i.e. the root tensor and the its right argument).