Quantum Computing - Exercises 6

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1. Consider Grover's algorithm as in Section 7 of the lecture notes.

   (i) Show that

   $$(2|\psi\rangle\langle\psi| - I)(\sum_x \alpha_x |x\rangle) = \sum_x (-\alpha_x + 2\langle\alpha\rangle)|x\rangle,$$

   where $\langle\alpha\rangle = \sum_x \alpha_x / N$.

   (ii) Explain why the operation $2|\psi\rangle\langle\psi| - I$ in Grover's algorithm is called *inversion about mean.* □

2. Show that, in Grover's algorithm, the action of $2|\psi\rangle\langle\psi| - I$ in the $|\sigma\rangle$ and $|\tau\rangle$ plane is reflection about $|\psi\rangle$.

3. Show that if we change the computational basis so that $|\sigma\rangle$ and $|\tau\rangle$ are basis elements, then the matrix representation of the Grover's transform G will be;

$$G_{\sigma,\tau} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix},$$

where $\theta/2$ is the angle between $|\sigma\rangle$ and $|\psi\rangle$.

4. In the Grover's algorithm, let $n = 2$ so that $N = 2^n = 4$, and let $M = 1$. The oracle f with $f(x) = 0$ for all $x \neq x_0$ and $f(x_0) = 1$ can be chosen from the four circuits in Figure 1, corresponding to $x_0 = 0, 1, 2, 3$ respectively from left to right.

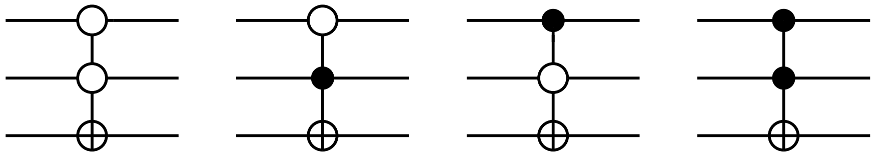

Figure 1: Four possible oracles

Show that the circuit in Figure 2 implements the operation G. How many iterates of G are needed to determine x_0?

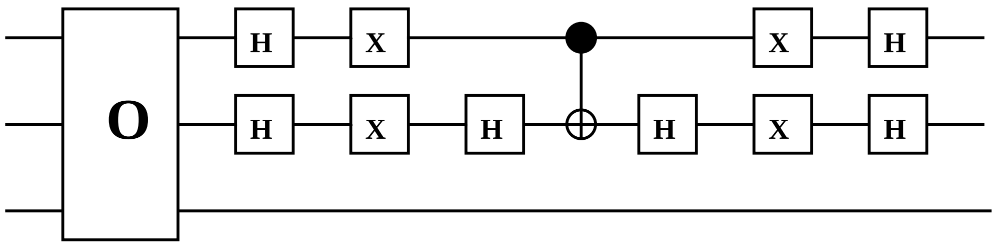

Figure 2: Circuit for G

5. What is the matrix for the quantum Fourier transform F when $n = 1$ and $n = 2$? Show that F is unitary for any n.