# Domain theory and Continuous Data Types
## Exercises 6

1.

(i) What is the boundary of $O(c,r)$? How about that of $C(c,r)$?

(ii) Show that the Cantor set is closed. What is its boundary?

2. Check that a subset $O \subseteq D$ of a dcpo $D$ is open if (i) $O$ is upward closed, i.e. $x \in O \;\&\; x \sqsubseteq y \;\Rightarrow\; y \in O$; and (ii) whenever $A \subseteq D$ is a directed subset with $\bigsqcup A \in O$, then $A \cap O \neq \emptyset$.

3.

(i) Check that in Euclidean spaces, if a sequence has a limit then the limit is unique. Show that a convergent sequence in a dcpo may have more than one limit in general.

(ii) Show that a closed subset $C$ of a topological space contains all its limit points, i.e. whenever $\lim_{n\to\infty} x_n = x$, with $x_n \in C$ for all $n \geq 0$, then $x \in C$.

(iii) Show by an example that membership in a closed set in Euclidean spaces is not semi-decidable.

4.

(i) Check that a function $f : \mathbb{R}^n \to \mathbb{R}^n$ is continuous with respect to the Euclidean topology iff it preserves limits of convergent sequences, i.e. iff whenever $\lim_{n\to\infty} x_n = x$ then $\lim_{n\to\infty} f(x_n) = f(x)$.

(ii) Check that a function $f : D \to E$ between dcpo's $D$ and $E$ is continuous with respect to the Scott topology iff $f$ is monotone and preserves the lubs of directed sets.

5. Show that the classical membership predicate of the unit disk $D \subset \mathbb{R}^n$ i.e. $\epsilon_D$ is not continuous at any point on the boundary of $D$. Show that $\epsilon'_S$ is continuous for any subset $S \subseteq \mathbb{R}^n$.