Quantum Computing - Exercises 7

Abbas Edalat

1.

(i) Work out the matrix for the quantum Fourier transform F and the network which implements it for $n = 3$.

(ii) Show that the inverse quantum Fourier transform is given by:

$$F^\dagger : |j\rangle \mapsto \frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} e^{-2\pi i jk/2^n} |k\rangle.$$

(iii) Work out the circuit for the inverse of the Fourier transform.

2.

(i) Find the eigenvalues and eigenvectors of the Pauli matrices, X, Y, Z. Locate the eigenvectors on the Bloch sphere.

(ii) Show that the eigenvalues of a unitary matrix have norm one.

(iii) Show that a change of basis does not alter the eigenvalues of a matrix. How do the eigenvectors alter with a change in the basis?

3. Diagonalize the three Pauli matrices and the Hadamard matrix.

4. Suppose the states $|u\rangle$ for $u \in T$ are eigenstates of U with eigenvalue $e^{2\pi i \phi_u}$. The phase estimation algorithm (Section 9 in the lecture notes) maps the normalized state

$$|0\rangle(\sum_{u \in T} d_u |u\rangle)$$

to the state

$$\sum_{u \in T} d_u |\hat{\phi}_u\rangle |u\rangle,$$

where the state $|\hat{\phi}_u\rangle$ gives a good estimate of ϕ_u. Show that with t chosen as

$$t = n + \left\lceil \log(2 + \frac{1}{2\epsilon}) \right\rceil,$$

the probability of measuring ϕ_u accurate to n bits in the output of the phase estimation algorithm is at least $|d_u|^2(1-\epsilon)$.