Domain theory and Continuous Data Types
Exercises 9

1. Let $1 = \{\perp\}$ be the one-point cpo. For each of the following domain equations $X \cong F(X)$, find the iterates $D_n = F^n(1)$, for $0 \leq n \leq 3$, and the embedding-projection pairs $(e_n, p_n) : D_n \lhd D_{n+1}$, for $0 \leq n \leq 2$, by drawing a diagram as in page 105 of the notes. How can we characterize D_∞, the solution of the domain equation, in each case?

(i) $X \cong 1_\perp \oplus X$.

(ii) $X \cong 1 + X$.

2. Consider the domain equation $D \cong F(D) = \Sigma_\perp \times D$, where $\Sigma = \{0, 1\}$, as in page 105 of the notes. Let $D_n = F^n(D_0)$ where $D_0 = \{\perp\}$.

(i) Check that $e_1 = F(e_0)$ and $p_1 = F(p_0)$, where e_0, e_1 and p_0, p_1 are as depicted on page 105.

(ii) Find D_n and D_{n+1}. How would you define the two maps $e_n : D_n \to D_{n+1}$ and $p_n : D_{n+1} \to D_n$ which give the embedding-projection pair $(e_n, p_n) : D_n \lhd D_{n+1}$?

(iii) Find D_∞. Which simple cpo is D_∞ isomorphic to? What are the functions constituting the isomorphism in the domain equation?