SOFTWARE ENGINEERING – DESIGN I

Tutorial 1: *Logic Revision and Basic use of schemas*

1. Consider the formula $\exists x \forall y(P(a,y) \rightarrow Q(f(y), x)$ and the following structure $X = \mathbf{N}$, $a=2$, $f(x) = x^2$, $P = \{(x,y) \mid x \leq y\}$ and $Q = \{(x,y) \mid y \text{ is a multiple of } x\}$.

   Is the above structure a model of the given formula? Explain your answer.

2. For each of the following, write down a schema whose models are as described.

   (a) A set with a transitive relation.
   (b) An odd natural number.
   (c) A set and a function from this set to the set of natural numbers, such that for each element of the set, the function gives a value within the interval [18,21].
   (d) An ordered list of integers.
   (e) Two sets and a finite partial function between them (expressed as a finite set of pairs (argument, result)).

   Note – Use of "$\mathbf{F}X$" as a sort: $\mathbf{F}X = \{Y \subseteq X \mid Y \text{ finite}\}$.

3. Suppose you're conducting a study on the heights of people. Write down a schema that includes

   (i) the overall Population of people,
   (ii) the function that shows their heights (in a whole number of mm and always less than 5m), and
   (iii) a finite Sample of people.

4. (i) Write a schema "FSet" whose model consists of both a finite set of integers and two consecutive members of this finite set.
   (ii) Write a schema "NewFSet" that includes both FSet and a second finite set of integers. This second set should contain all the elements of the finite set specified in FSet and at least one more.
   (iii) Write NewFSet in full without any schema inclusion.

5. What are the models of the following schema, and how many are they?

   _____ Ind _____

   $P \subseteq \mathbf{N}$

   $P(0)$
   $\forall n{:}\mathbf{N}.\ (P(n) \rightarrow P(n+1))$

6. (*For the more adamant*) Repeat question 4 for the following schema. What is a simple way of saying that $ys \in P(xs)$? Can you explain why?

   _____ [X] _____

   P: seq $X \rightarrow \mathbf{F}$seq X

   $P([]) = \{[]\}$
   $\forall x{:}\ X.\ \forall xs{:}\ \text{seq } X.\ P(x{:}xs) = \{us{+}{+}[x]{+}{+}vs \mid us{+}{+}vs \in P(xs)\}$

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Tutorial 1 ANSWERS: *Logic Revision and Basic use of schemas*

A1. The given structure is not a model of the given formula. Interpreting the formula in this model, would give us that the formula is true if there exist a natural number that is a multiple of the square of every natural number greater than 2. This is clearly not true.

A2.

(a)

Transitivity[X]

$R \subseteq X \times X$

$\forall x,y,z:X.\ (R(x,y) \wedge R(y,z) \rightarrow R(x,z))$

(b)

OddNumber

$n: \mathbf{N}$

$\exists\ h:\mathbf{N}.\ (n = (2*h) + 1)$

(c)

Range[X]

$f: X \rightarrow \mathbf{N}$

$\forall x: X.\ ((f(x) \geq 18) \wedge (f(x) \leq 21))$

(d)

Ordered List

$xs: \text{seq } \mathbf{Z}$

$\forall\ us,vs: \text{seq } Z.\ \forall x,y: Z.\ (xs = us \mathbin{++} [x,y] \mathbin{++} vs \rightarrow x \leq y)$

(e)

Partial Function[X,Y]

$f: \mathbf{F}(X \times Y)$

$\forall x:X.\ \forall y_1,y_2:Y.\ ((x,y_1) \in f \wedge (x,y_2) \in f \rightarrow y_1 = y_2)$

A3.

Test[Population]

$height: Population \rightarrow \mathbf{N}$

$Sample: \mathbf{F}Population$

$\forall x: Population.\ height(x) < 5000$

A4.

(i)

FSet

$P: \mathbf{FZ}$

$n,m: \mathbf{Z}$

$n \in P$

$m \in P$

$m = n+1 \vee n = m+1$

(ii)

NewFSet

FSet

$P': \mathbf{FZ}$

$\forall x:Z.\ (x \in P \rightarrow x \in P')$

$\exists x:Z.\ (x \in P' \wedge x \notin P)$

(iii)

NewFSetFull

$P: \mathbf{FZ}$

$n,m: \mathbf{Z}$

$P': \mathbf{FZ}$

$n \in P$

$m \in P$

$m = n+1 \vee n = m+1$

$\forall x:Z.\ (x \in P \rightarrow x \in P')$

$\exists x:Z.\ (x \in P' \wedge x \notin P)$

A5. There is just one model: $\mathbf{N}$ has to be interpreted as the natural numbers, and by induction the axioms imply that P is the predicate that is true for all natural numbers.

A6.

(a) A model of the schema is a set together with a function P that is effectively defined by taking the axioms as a recursive definition. There are as many models as possible carrier sets X.

(b)Whatever X is, $ys \in P(xs)$ iff ys is a permutation of xs. We can prove this by list induction. It is certainly true if xs = [], and for x:xs we have that ys is a permutation of x:xs iff it can be written in the form us++[x]++vs with us++vs a permutation of xs, i.e. (by the induction hypothesis) with $us \mathbin{++} vs \in P(xs)$.