Optical Flow Estimation with Uncertainties through Dynamic MRFs

Ben Glocker[1,2], Nikos Paragios[1], Nikos Komodakis[1,3], Georgios Tziritas[3], Nassir Navab[2]

[1] GALEN Group, Mathématiques Appliquées aux Systèmes (MAS), Ecole Centrale Paris, France
[2] Computer Aided Medical Procedures (CAMP), Technische Universität München, Germany
[3] Computer Science Department, University of Crete, Greece

Flow Estimation through MRFs

Brightness Constancy Assumption

$$I(\mathbf{x},f)=I(\mathbf{x}+\mathcal{D}(\mathbf{x}),f+1)$$

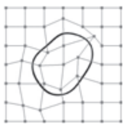
Flow Estimation as Energy Minimization Problem

$$E(\mathcal{D})=\int_\Omega \underbrace{|I(\mathbf{x},f)-I(\mathbf{x}+\mathcal{D}(\mathbf{x}),f+1)|}_{\rho(\mathbf{x})\quad(\mathrm{DataTerm})}+\underbrace{\phi(\nabla\mathcal{D}(\mathbf{x}))}_{(\mathrm{SmoothnessTerm})}\;d\mathbf{x}$$

1. Dimensionality Reduction

Transformation Model based on Linear Combination of Control Points

$$\mathcal{D}(\mathbf{x})=\sum_{\mathbf{p}\in G}\eta(|\mathbf{x}-\mathbf{p}|)\,\mathcal{C}(\mathbf{p})$$



Reformulation of Energy Terms [1]

$$E_{\mathrm{data}}(\mathcal{D})=\frac{1}{|G|}\sum_{\mathbf{p}\in G}\int_\Omega\hat\eta(|\mathbf{x}-\mathbf{p}|)\,\rho(\mathbf{x})d\mathbf{x}$$

$$E_{\mathrm{smooth}}(\mathcal{D})=\frac{1}{|G|}\sum_{\mathbf{p}\in G}\int_\Omega\hat\eta(|\mathbf{x}-\mathbf{p}|)\,(|\partial_x\mathcal{D}(\mathbf{x})|+|\partial_y\mathcal{D}(\mathbf{x})|)\,d\mathbf{x}$$

Back-Projection Function

$$\hat\eta(|\mathbf{x}-\mathbf{p}|)=\frac{\eta(|\mathbf{x}-\mathbf{p}|)}{\int_\Omega\eta(|\mathbf{y}-\mathbf{p}|)d\mathbf{y}} \qquad \hat\eta(|\mathbf{x}-\mathbf{p}|)=\begin{cases}1, & \text{if } \eta(|\mathbf{x}-\mathbf{p}|)>0\\ 0 & \text{if } \eta(|\mathbf{x}-\mathbf{p}|)=0\end{cases}$$

Pixel-wise Measures — Statistical Measures

2. Discrete MRF Formulation

Discrete Set of Labels and Quantized Version of Displacement Space

$$L=\{l^1,...,l^n\} \qquad \Theta=\{\mathbf{d}^1,...,\mathbf{d}^n\}$$

Energy of Discrete Labeling as Sum of Unary and Pairwise Potential Functions

$$E_{\mathrm{MRF}}(l)=\sum_{\mathbf{p}\in G}V_{\mathbf{p}}(l_{\mathbf{p}})+\sum_{\mathbf{p}\in G}\sum_{\mathbf{q}\in\mathcal{N}(\mathbf{p})}V_{\mathbf{pq}}(l_{\mathbf{p}},l_{\mathbf{q}})$$

Definition of Potential Functions

$$\underbrace{V_{\mathbf{p}}(l_{\mathbf{p}})\approx\int_\Omega\hat\eta(|\mathbf{x}-\mathbf{p}|)\,\rho(\mathbf{x})d\mathbf{x}}_{\text{Fast Approximation Scheme}} \qquad V_{\mathbf{pq}}(l_{\mathbf{p}},l_{\mathbf{q}})=\lambda\left|(\mathcal{C}(\mathbf{p})+\mathbf{d}^{l_{\mathbf{p}}})-(\mathcal{C}(\mathbf{q})+\mathbf{d}^{l_{\mathbf{q}}})\right|$$

Optimization through Fast-PD Algorithm [2]

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Uncertainty Estimation for Dynamic Label Set Adjustment

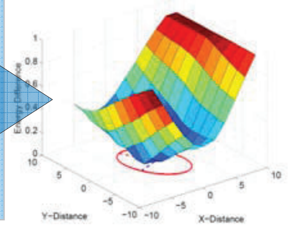
Min-marginal Energy Computation [2,3] for Multi-Labeling Solutions

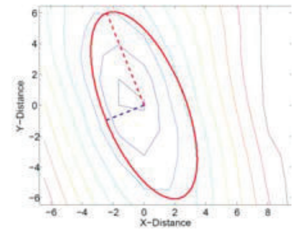
$$\mu_p(\alpha)=\min_{l:l_p=\alpha}E(l) \qquad V^{\mathrm{new}}_{\mathbf{p}}(l_{\mathbf{p}})=\begin{cases}V_{\mathbf{p}}(\alpha), & \text{if } l_{\mathbf{p}}=\alpha\\ \infty, & \text{if } l_{\mathbf{p}}\neq\alpha\end{cases}$$

Iterative Computation of Uncertainty Map for each Control Point using Fast-PD for Dynamic MRFs [2]

$$U_p(\alpha)=\frac{\exp^{-\mu_p(\alpha)}}{\sum_l\exp^{-\mu_p(l)}}$$

Local Adjustment of Discrete Displacement Space according to Covariance of Uncertainty Map





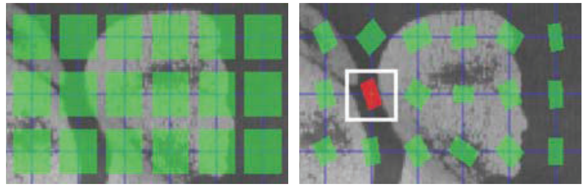
Experiments

Matching Criteria combining Photometrical and Geometrical Features

$$\rho_{\mathrm{CC+GIP}}=(1-\gamma)\left(1-\left|\frac{\sum_\Omega(a-\bar a)(b-\bar b)}{\sqrt{\sum_\Omega(a-\bar a)^2\sum_\Omega(b-\bar b)^2}}\right|\right)+\gamma\sum_\Omega\left|\frac{\nabla a}{|\nabla a|}\cdot\frac{\nabla b}{|\nabla b|}\right|$$

Evaluation on Opical Flow Data Base [4]

- Training Data with Ground Truth for Parameter Setting
- Evaluation Data with Remote Error Computation



Average Angular Error	Army			Mequon			Schefflera			Wooden			Grove			Urban			Yosemite			Teddy		
	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext
Dynamic MRF	4.95	15.0	4.20	3.71	15.3	2.58	6.77	17.8	3.26	4.50	23.7	2.62	3.95	4.63	4.65	7.67	19.1	7.20	3.63	5.29	4.62	8.32	17.8	8.16
LP-Registration	7.36	16.8	6.30	3.94	13.8	3.00	7.33	17.8	4.43	5.54	24.5	3.57	4.24	4.56	5.63	10.5	21.4	9.57	4.54	5.48	3.95	8.15	17.9	7.82
Black & Anandan 2	7.83	18.7	6.41	9.70	21.9	8.60	13.7	23.7	18.1	10.9	30.0	9.44	4.43	5.23	4.94	7.95	18.2	6.51	2.61	4.44	2.15	8.58	14.3	8.54
2D-CLG	10.1	22.6	7.59	9.84	16.9	11.1	16.9	28.2	18.8	14.1	31.1	13.1	3.66	4.25	4.41	6.69	22.2	6.96	1.76	3.14	1.46	6.29	12.9	5.81
Horn & Schunck	8.01	19.9	8.38	9.13	23.2	7.71	14.2	25.9	14.6	12.4	30.6	11.3	4.44	5.27	4.59	8.25	25.8	8.77	4.01	5.41	1.95	9.16	17.5	8.86
Black & Anandan	8.93	18.5	9.99	12.9	22.4	13.3	15.8	25.9	18.3	13.2	31.8	12.0	5.69	6.35	7.77	9.37	18.8	9.02	3.10	4.88	3.96	13.4	18.3	15.1
Pyramid LK	13.9	20.9	21.4	24.1	23.1	30.2	20.9	29.5	21.9	22.2	34.6	25.0	18.7	22.9	19.9	21.9	26.2	23.5	6.41	7.02	10.8	25.6	31.5	34.5
MediaPlayerTM	18.3	30.8	15.0	17.7	29.2	17.4	19.9	32.7	21.6	26.3	45.9	25.9	7.23	6.95	10.2	19.4	32.9	19.3	12.7	18.7	17.2	17.4	22.9	20.7

References

[1] Glocker et al. Inter and Intra-Modal Deformable Registration: Continuous Deformations Meet Efficient Optimal Linear Programming. IPMI 2007
[2] Komodakis et al. Fast, Approximately Optimal Solutions for Single and Dynamic MRFs. CVPR 2007
[3] Kohli & Torr. Measuring Uncertainty in Graph Cut Solutions: Efficiently Computing Min-marginal Energies using Dynamic Graph Cuts. ECCV 2006
[4] Baker et al. A database and evaluation methodology for optical flow. ICCV 2007