Dense Registration with Deformation Priors

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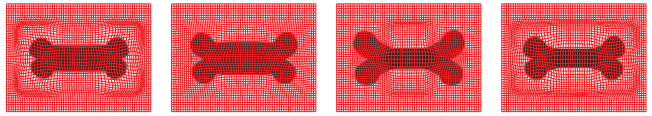
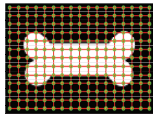
Estimation of Deformation Priors

Set of N training examples (e.g. obtained through dense registration)



1. Dimensionality Reduction

Parametrization of a displacement field by M control points (e.g. FFDs)

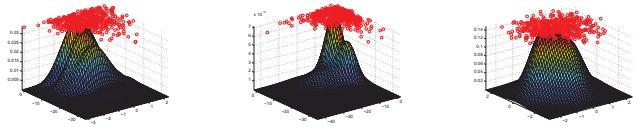


$$D(\mathbf{x}) = \sum_{i=1}^{M} \eta(\mathbf{x})\ \mathbf{d}_i$$

$\eta(\mathbf{x})$: basis functions (e.g. cubic B-splines)
$\mathbf{d}_i$: control point displacement

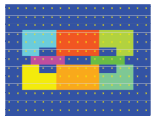
2. Displacement Distributions

Estimation of M distributions w.r.t. to the training data (e.g. Gaussian Mixture Models)



3. Control Point Clustering

Grouping of control points w.r.t. their displacement distributions



$$\min_{K,C,A} \sum_{i=1}^{M} \xi(\kappa_{\mathbf{p}_i}, \kappa_{\mathbf{c}_{a_i}}) + \sum_{k=1}^{K} f(\mathbf{c}_k) \qquad \xi(\kappa_i, \kappa_j) = \underbrace{-\log \int_{-\infty}^{\infty} \sqrt{\kappa_i(\mathbf{x})\kappa_j(\mathbf{x})}\, d\mathbf{x}}_{\text{Bhattacharyya Divergence}}$$

K: number of clusters
C: cluster centers
A: element assignments
κ: probability distribution
ξ: distance function
f: cluster creation penalty

See reference [1]

Conventional Registration using MRFs

Second-order MRF labeling energy

$$E_{\text{mrf}}(\mathbf{l}) = \sum_{\mathbf{p}\in P} V_{\mathbf{p}}(l_{\mathbf{p}}) + \lambda \sum_{(\mathbf{p},\mathbf{q})\in S} V_{\mathbf{pq}}(l_{\mathbf{p}}, l_{\mathbf{q}})$$

$\mathbf{l}$: discrete labeling
P: set of control points
S: neighborhood system

Unary terms encoding similarity measure

$$V_{\mathbf{p}}(l_{\mathbf{p}}) = \int_{\Omega} \hat{\eta}(\mathbf{x}) \cdot \phi\left(I(\mathbf{x}), J(\mathbf{x} + D^{t-1}(\mathbf{x}) + \mathbf{d}_{l_{\mathbf{p}}})\right) d\mathbf{x}$$

$\hat{\eta}$: influence function
ϕ: similarity measure

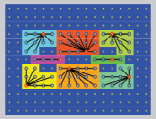
Pairwise terms encoding regularization (on a grid-like neighborhood system)

$$V_{\mathbf{pq}}(l_{\mathbf{p}}, l_{\mathbf{q}}) = \left((\mathbf{d}_{\mathbf{p}}^{t-1} + \mathbf{d}_{l_{\mathbf{p}}}) - (\mathbf{d}_{\mathbf{q}}^{t-1} + \mathbf{d}_{l_{\mathbf{q}}})\right)^2$$

See reference [2]

MRF Registration with Priors

Local deformation prior for each cluster (*intra-cluster edges*)

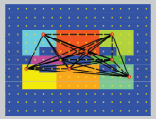


$$\psi_k^{\text{local}}(D) = \prod_{\mathbf{p},\mathbf{c}_k\in P_k, \mathbf{c}_k\in C, \mathbf{p}\neq\mathbf{c}_k} \kappa_{\mathbf{pc}_k}(\|\mathbf{d}_{\mathbf{p}} - \mathbf{d}_{\mathbf{c}_k}\|)$$

Corresponding MRF potential

$$V_{\mathbf{pc}_k}(l_{\mathbf{p}}, l_{\mathbf{c}_k}) = -\log\left(\kappa_{\mathbf{pc}_k}(\|(\mathbf{d}_{\mathbf{p}}^{t-1} + \mathbf{d}_{l_{\mathbf{p}}}) - (\mathbf{d}_{\mathbf{c}_k}^{t-1} + \mathbf{d}_{l_{\mathbf{c}_k}})\|)\right)$$

Global deformation prior on cluster centers (*inter-cluster edges*)



$$\psi^{\text{global}}(D) = \prod_{\mathbf{c}_i,\mathbf{c}_j\in C, i\neq j} \kappa_{\mathbf{c}_i\mathbf{c}_j}(\|\mathbf{d}_{\mathbf{c}_i} - \mathbf{d}_{\mathbf{c}_j}\|)$$

Corresponding MRF potential

$$V_{\mathbf{c}_i\mathbf{c}_j}(l_{\mathbf{c}_i}, l_{\mathbf{c}_j}) = -\log\left(\kappa_{\mathbf{c}_i\mathbf{c}_j}(\|(\mathbf{d}_{\mathbf{c}_i}^{t-1} + \mathbf{d}_{l_{\mathbf{c}_i}}) - (\mathbf{d}_{\mathbf{c}_j}^{t-1} + \mathbf{d}_{l_{\mathbf{c}_j}})\|)\right)$$

→ ***Registration with priors by changing the MRF topology and replacement of potentials***

Novel MRF energy

$$E_{\text{mrf}}(\mathbf{l}) = \underbrace{\sum_{\mathbf{p}\in P} V_{\mathbf{p}}(l_{\mathbf{p}})}_{\text{Data Costs}} + \lambda \underbrace{\left(\sum_{\mathbf{c}_i,\mathbf{c}_j\in C} V_{\mathbf{c}_i\mathbf{c}_j}(l_{\mathbf{c}_i}, l_{\mathbf{c}_j}) + \sum_{k=1}^{K} \sum_{\mathbf{p},\mathbf{c}_k\in P_k} V_{\mathbf{pc}_k}(l_{\mathbf{p}}, l_{\mathbf{c}_k})\right)}_{\text{Deformation Prior Costs}}$$

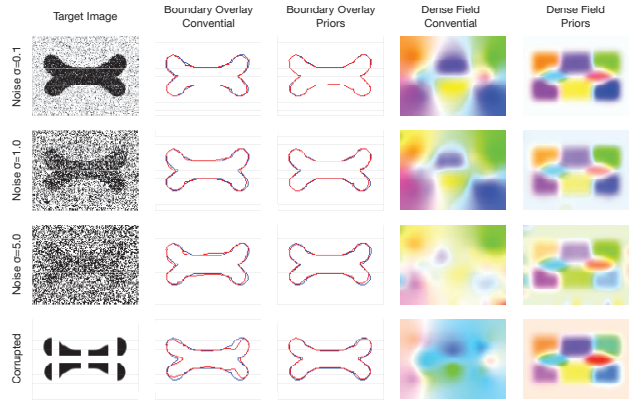
Experimental Validation

Synthetic data


Initial Alignment

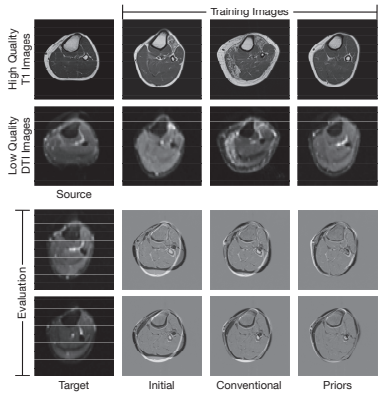
Target	Conventional	Priors
Noise σ=0.1	0.42 (± 0.11)	0.16 (± 0.06)
Noise σ=1.0	0.73 (± 0.18)	0.42 (± 0.15)
Noise σ=5.0	1.17 (± 0.26)	0.79 (± 0.25)
Corrupted	1.52 (± 0.49)	0.69 (± 0.12)

Table: Average Boundary Distance



Real data

- A set of high quality T1 images is used for training and learning of deformation priors
- The corresponding low quality DTI images are then registered to a remaining set of evaluation images
- The DTI registration is performed using the proposed MRF registration with deformation priors estimated from the T1 registrations
- The registration results are visually compared to conventional MRF registration



References:
[1] Komodakis N. et al. **Clustering via lp-based stabilities**. NIPS 2008
[2] Glocker B. et al. **Dense image registration through mrfs and efficient linear programming**. MedIA 12(6) 2008

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