

# TriangleFlow: Optical Flow with Triangulation-based Higher-Order Likelihoods

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## Properties of Matching Costs for Optical Flow

### \* Patch based: Discriminative, Constant flow assumption

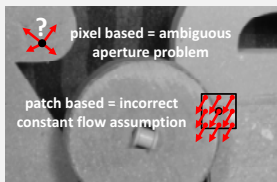
- considers multiple pixels simultaneously
- incorrect costs at flow boundaries

### \* Pixel based: Ambiguous, Valid

- ill-posed problem
- assigns correct cost to the label assignment

### \* Higher order: Discriminative, Valid

- considers multiple pixels simultaneously
- assigns correct cost to the label assignment



## Higher-Order Likelihoods for Optical Flow

### \* Tractable?

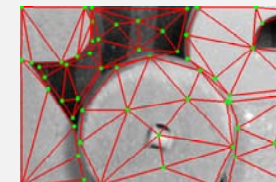
- Clique of K pixels has a label space of size  $|L|^K$

### \* Our assumption: Optical flow is piecewise affine

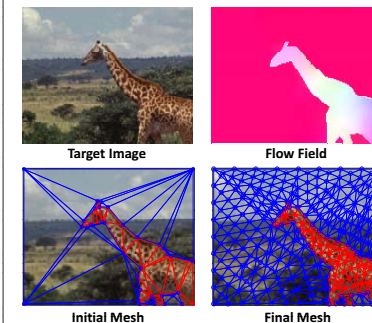
- mainly solid objects which translate, rotate, or scale
- non-solids (e.g. textiles) can be modeled by local affine motions

### \* Our solution: Complexity reduction via triangulation

- triangulations are adaptive and flexible
- each triangle constitutes a local affine warp on a sub-image



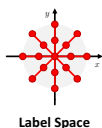
## Giraffe Sequence



## Random Field Energy: $E(x) = \sum_{c \in C} \psi_c(x_c)$

### \* Optimization via hierarchical fusion moves

- based on QPBO and higher-order clique reduction
- each MAP labeling is an update on the triangle configuration
- nodes correspond to triangle control points
- labels correspond to 2D displacement vectors
- iterative mesh and label space refinement
- sparse star-like search space discretization



Label Space

### Algorithm: TriangleFlow

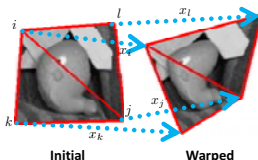
```

input: Images I, J
output: Flow field F
1: F := estimateInitialFlow(I, J);
2: M := estimateMultiLayerMesh(I, F);
3: L := initializeLabelSpace();
4: while continue
5:   ΔF := fusionMoves(I, J, F, M, L);
6:   if (E(F+ΔF) < E(F))
7:     F := F+ΔF;
8:   else if (L_count < L_max)
9:     L := refineLabelSpace();
10:    L_count++;
11:  else if (M_count < M_max)
12:    M := refineMesh();
13:    L_count=0;
14:    M_count++;
15:  else continue := false;
16: end
    
```

### \* Likelihood encoded on triple cliques

- each clique labeling is a local affine warp
- one energy term per triangle
- evaluates the cost between sub-image  $I_{ijk}$  and  $J_{ijk}$  exactly
- photo-consistent matching cost: correlation coefficient

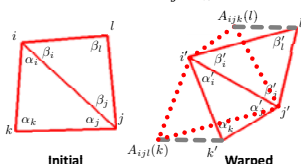
$$\psi_{ijk}(x_i, x_j, x_k) = 1 - \frac{\text{cov}(I_{ijk}, J_{ijk})}{\sigma_{I_{ijk}} \sigma_{J_{ijk}}}$$



Initial Warped

### \* Motion prior on triple cliques: ADP

- triangle-based: angle deviation penalty (ADP)
- penalizes difference between initial angles  $(\alpha_i, \alpha_j, \alpha_k)$  and warped angles  $(\alpha'_i, \alpha'_j, \alpha'_k)$
- allows translation, rotation, and scaling of triangle  $(i, j, k)$



Initial Warped

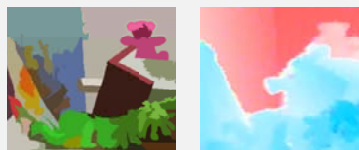
### \* Motion prior on quadruple cliques: NAMP

- rectangle-based: non-affine motion penalty (NAMP)
- penalizes warps  $A_{ijk}$  and  $A_{ijl}$  of neighboring triangles
- allows affine motions for the rectangle  $(i, j, k, l)$

## Layered Representation

### \* Goal: Determine layers of consistent motion

### \* Input: Over-segmentation + Initial flow field

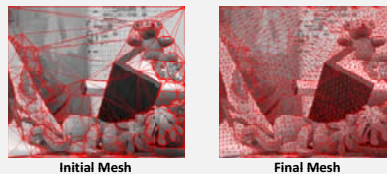


### \* Spectral clustering: Fixed to 15 clusters

### \* Output: Segments with similar affine motion

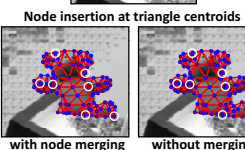
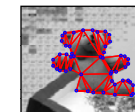


### \* Multi-Layer: One triangulation per segment



Initial Mesh Final Mesh

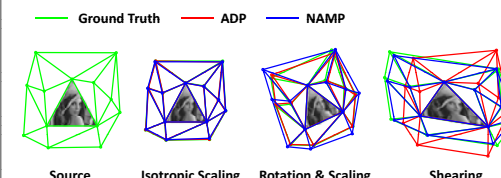
## Mesh Refinement



with node merging without merging

## Motion Penalty: ADP versus NAMP

### \* Evaluation: Comparison of lowest energy configurations for affine transformations under different priors



## Evaluation on the Middlebury Dataset

### \* TriangleFlow: Sharp motion boundaries, high-quality flow fields, takes up to 4200 seconds

### \* Multi-Layer: 15 motion layers, explicit occlusion handling in overlap areas

### \* Mesh Refinement: coarse-to-fine strategy, 4 levels, > 10,000 triangles on finest level



Single-Layer Regular Mesh EP 0.14 / AE 5.22

Multi-Layer TriangleFlow EP 0.11 / AE 4.12

Single-Layer Regular Mesh EP 1.00 / AE 6.70

Multi-Layer TriangleFlow EP 0.63 / AE 3.02