Algorithms for Optimal Decisions
Tutorial 5
Questions

Active Set Algorithm

1. Given an initial feasible point x_0 identify the set $\mathcal{I}_0$.

2. Set $k = 0$.

3. Solve the following **QP** problem to obtain $d_k = \bar{x} - x_k$:

$$\begin{aligned} \min_x \quad & a^t x + \frac{1}{2} x^t Q x \\ s.t. \quad & \mathcal{H}_k^t x = h_k. \end{aligned} \tag{1}$$

4. If $d_k = 0$ go to step 9.

5. Determine τ_k as:

$$\tau_k = \min_{j \notin \mathcal{I}_k} \{1, \frac{h^j - (\nabla h^j, x_k)}{(\nabla h^j, d_k)} \quad | \quad (\nabla h^j, d_k) > 0\}. \tag{2}$$

6. Set $x_{k+1} = x_k + \tau_k d_k$.

7. If $\tau_k = 1$ go to 9.

8. Add the new constraint determined in (2) to the active set $\mathcal{I}_k$ to form $\mathcal{I}_{k+1}$. Go to 3.

9. Compute μ_{k+1}:

$$\mu_{k+1} = -(\mathcal{H}_k^t \mathcal{H}_k)^{-1} \mathcal{H}_k^t [a + Q x_k + Q d_k]. \tag{3}$$

10. Determine the minimum element $\mu_{\min}$ of μ_{k+1}.

11. If $\mu_{\min} < 0$ drop the i–th constraint from the active set $\mathcal{I}_k$ to obtain $\mathcal{I}_{k+1}$. Set $k = k + 1$. Go to 3.

12. Otherwise x_{k+1} is optimal. Stop.

Comment If a new point $\bar{x}$ is feasible for the full set of constraints then $\tau = 1$ so we do not need to do step 5. We go to step 9 and check whether there are some constraints that we may drop.

Exercise 1

Solve the following **QP** *using the active set method:*

$$\begin{aligned} \min_x \quad f(x) &= 2x_1^2 + x_1x_2 + x_2^2 - 12x_1 - 10x_2 \\ s.t. \quad h_1(x) &= x_1 + x_2 - 4 \leq 0 \\ h_2(x) &= -x_1 \leq 0 \\ h_3(x) &= -x_2 \leq 0. \end{aligned} \tag{4}$$

Starting point : $x^{(0)} = (x_1^{(0)}, x_2^{(0)}) = (0, 0)$.

Exercise 2 *Solve the following problem by using the active set method and taking*
$x^{(0)} = (x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (0, 0, 1)$ *as a starting point*

$$\begin{aligned} \min_x f(x) &= x_1^2 + 2x_2^2 + 3x_3^2 \\ s.t. \quad & x_1 + x_2 + x_3 - 1 \geq 0 \\ & x_1, x_2, x_3 \geq 0. \end{aligned} \tag{5}$$